Base Logics in Argumentation

Anthony Hunter

Dept of Computer Science, UCL, London

Base logics for argumentation

In general, we can regard a logical argument as a tuple $\langle \Phi, \alpha \rangle$ where

- Φ is the support
- α is the claim
- Φ entails α

In addition, we may require that Φ is consistent and/or Φ is minimal for entailing α.

So arguments are constructed using some **base logic** $\vdash_x$ such as classical logic, defeasible logic, contrapositive logic, temporal logic, description logic, etc.

Classical logic for argumentation

Let Δ be set of formulas in classical logic

An **argument** is a pair $\langle \Phi, \alpha \rangle$ such that

1. $\Phi \not\vdash \bot$
2. $\Phi \vdash \alpha$
3. Φ is a minimal subset of Δ satisfying 2

For $\Delta = \{\neg\neg a, \neg b \rightarrow \neg a, \neg b \vee (c \wedge d), b \wedge c \wedge \neg b, \neg f \rightarrow g \vee h\}$, the following is an argument

$$\langle \{\neg\neg a, \neg b \rightarrow \neg a, \neg b \vee (c \wedge d)\}, e \rightarrow d \rangle$$

Defeasible logics for argumentation

Let Δ be the union of a set of rules and a set of literals. The defeasible logic consequence relation $\vdash_d$ is defined as follows.

$\Delta \vdash_d \psi$ iff there is a sequence of literals $\alpha_1, \ldots, \alpha_n$

such that ψ is α_n and for each $\alpha_i \in \{\alpha_1, \ldots \alpha_n\}$

either α_i is a literal in Δ

or there is a $\beta_1 \wedge \ldots \wedge \beta_j \rightarrow_k \alpha_i \in \Delta$

and $\{\beta_1, \ldots, \beta_j\} \subseteq \{\alpha_1, \ldots, \alpha_{i-1}\}$

Let $\Delta = \{p, \neg q, p \rightarrow_1 \neg r, \neg q \wedge \neg r \rightarrow_2 s, s \rightarrow_1 t, p \wedge t \rightarrow_2 u\}$.
Therefore $\Delta \vdash_d u$ where the derivation is $p, \neg r, \neg q, s, t, u$.

Defeasible logics for argumentation

For $\Delta = \{p, \neg q, p \rightarrow_1 \neg r, \neg q \wedge \neg r \rightarrow_2 s, s \rightarrow_1 t, p \wedge t \rightarrow_2 u\}$,
the following is an argument in defeasible logic programming.

$$\langle \{p, \neg q, p \rightarrow_1 \neg r, \neg q \wedge \neg r \rightarrow_2 s, s \rightarrow_1 t, p \wedge t \rightarrow_2 u\}, u \rangle$$

For $\Delta = \{p, \neg q, s, p \rightarrow \neg r, \neg q \wedge \neg r \wedge s \rightarrow t, t \wedge p \rightarrow u, v\}$,
the following is an argument in assumption-based argumentation.

$$\langle \{p, \neg q, s, p \rightarrow \neg r, \neg q \wedge \neg r \wedge s \rightarrow t\}, t \rangle$$

The are limitations of defeasible logic. Consider $\Delta = \{a \rightarrow b, \neg a \rightarrow b\}$.
Hence, $\Delta \vdash_c b$, but $\Delta \not\vdash_d b$.

Defeasible logics for argumentation

Let Δ be the union of a set of enhanced defeasible rules and a set of strong literals. The enhanced consequence relation $\vdash_e$ is defined as follows.

$\Delta \vdash_e \psi$ iff there is a sequence of literals $\alpha_1, \ldots, \alpha_n$
such that ψ is α_n and for each $\alpha_i \in \{\alpha_1, \ldots \alpha_n\}$
either α_i is a strong literal in Δ
or there is a $\gamma_0 \wedge \ldots \wedge \gamma_m \wedge \beta_0 \wedge \ldots \wedge \beta_n \rightarrow_k \delta \in \Delta$
and $\{\gamma_0, \ldots, \gamma_m\} \subseteq \{\alpha_1, \ldots, \alpha_{i-1}\}$

Let $\Delta = \{p, p \rightarrow_1 \neg r, \neg r \wedge \sim q \rightarrow_2 s, s \rightarrow_1 t, p \wedge t \rightarrow_1 u\}$.
Therefore $\Delta \vdash_d u$ where the derivation is $p, \neg r, s, t, u$.

$$\langle \{p, p \rightarrow_1 \neg r, \neg r \wedge \sim q \rightarrow_2 s, s \rightarrow_1 t, p \wedge t \rightarrow_1 u\}, u \rangle$$

Contrapositive logic for argumentation

Let Δ be a set of rules and literals. The defeasible logic consequence relation $\vdash_f$ is defined as follows.

$\Delta \vdash_f \psi$ iff there is a sequence of literals $\alpha_1, \ldots, \alpha_n$
such that ψ is α_n and for each $\alpha_i \in \{\alpha_1, \ldots \alpha_n\}$
either α_i is a literal in Δ
or there is a $\beta_1 \wedge \ldots \wedge \beta_j \rightarrow_k \alpha_i \in \Delta \cup \mathsf{Contrapositives}(\Delta)$
and $\{\beta_1, \ldots, \beta_j\} \subseteq \{\alpha_1, \ldots, \alpha_{i-1}\}$

Let $\Delta = \{q, \neg r, p \wedge q \rightarrow r, \neg p \rightarrow u\}$.
So Contrapositives(Δ) = $\{\neg r \wedge q \rightarrow \neg p, p \wedge \neg r \rightarrow \neg q, \neg u \rightarrow p\}$.
Therefore, $\Delta \vdash_f u$, where the derivation is $q, \neg r, \neg p, u$.

Metatheorems of the consequence relation

$\Delta \cup \{\alpha\} \vdash_x \alpha$	(Reflexivity)
$\Delta \cup \{\alpha\} \vdash_x \alpha$ if α is a literal	(Literal reflexivity)
$\Delta \cup \{\beta\} \vdash_x \gamma$ if $\Delta \cup \{\alpha\} \vdash_x \gamma$ and $\vdash \alpha \leftrightarrow \beta$	(Left logical equivalent)
$\Delta \vdash_x \alpha$ if $\Delta \vdash_x \beta$ and $\vdash \beta \rightarrow \alpha$	(Right weakening)
$\Delta \vdash_x \alpha \wedge \beta$ if $\Delta \vdash_x \alpha$ and $\Delta \vdash_x \beta$	(And)
$\Delta \cup \{\alpha\} \vdash_x \beta$ if $\Delta \vdash_x \beta$	(Monotonicity)
$\Delta \vdash_x \beta$ if $\Delta \vdash_x \alpha$ and $\Delta \cup \{\alpha\} \vdash_x \beta$	(Cut)
$\Delta \vdash_x \alpha \rightarrow \beta$ if $\Delta \cup \{\alpha\} \vdash_x \beta$	(Conditionalization)
$\Delta \cup \{\alpha\} \vdash_x \beta$ if $\Delta \vdash_x \alpha \rightarrow \beta$	(Deduction)
$\Delta \cup \{\alpha\} \vdash_x \beta$ if $\Delta \cup \{\neg\beta\} \vdash_x \neg\alpha$	(Contraposition)
$\Delta \cup \{\alpha \vee \beta\} \vdash_x \gamma$ if $\Delta \cup \{\alpha\} \vdash_x \gamma$ and $\Delta \cup \{\beta\} \vdash_x \gamma$	(Or)

Inconsistency tolerance

- The consequence relation $\vdash_x$ is **trivializable** iff for all Δ, there is an atom α such that if $\Delta \vdash \alpha$ and $\Delta \vdash \neg\alpha$ then $\Delta \vdash_x \beta$ for all atoms β.
- A formula α is pure w.r.t. Δ iff $\mathsf{Atoms}(\Delta) \cap \mathsf{Atoms}(\{\alpha\}) \neq \emptyset$.
  A consequence relation $\vdash_x$ is **pure** iff for all α and Δ, if $\Delta \vdash_x \alpha$, then α is pure w.r.t. Δ.

Classical logic is trivializable and not pure, whereas defeasible logics and contrapositive logic are pure and not trivializable.

Even though most definitions for a logical argument assume consistency of premises, we may wish to allow paraconsistent inferences from inconsistent premises

Decision problems

	Entailment	Consistency checking
Classical	coNPC	NPC
Contrapositive	coNPC	NPC
Defeasible	P	P

For defeasible logic, there are polynomial time algorithms (see Mahler TPLP 2004)

Comparing base logics

- Meta-theorems of the consequence relation

  defeasible $<$ contrapositive $<$ classical

- Inferential strength of the consequence relation

  defeasible $<$ contrapositive $<$ classical

- Paraconsistent properties of the consequence relation

  defeasible & contrapositive are tolerant, whereas classical is not tolerant

- Complexity of entailment and consistency decision problems

  defeasible is tractable, whereas contrapositive & classical are not tractable

Each base logic has strengths and weaknesses for applications.

Combined logics for argumentation

Some formalisms can be viewed as a composition of logics (called bilogics).

- Some defeasible logic formalisms (e.g. defeasible logic programming, extended logic programming, etc) which involve multiple implication symbols such as for "defeasible" and for "strict" rules.
- Some hybrid formalisms for using defeasible logic with ontologies (e.g. a variant of defeasible logic programming in which conditions of defeasible rules can be evaluated by sub-contract to an ontology).

Combined logics for argumentation

Some logic-based approaches involve a combination of logics ($\vdash_1$,$\vdash_2$,...) for the definition of entailment ($\vdash$) when generating arguments.

$$(\Delta_1, \Delta_2) \vdash \alpha \text{ iff}$$
$$(\Delta_1, \Delta_2) \vdash \beta_1 \text{ and } \ldots (\Delta_1, \Delta_2) \vdash \beta_n$$
$$\text{and } (\Delta_1 \cup \{\beta_1, \ldots, \beta_n\} \vdash_1 \alpha \text{ or } \Delta_2 \cup \{\beta_1, \ldots, \beta_n\} \vdash_2 \alpha)$$

A combined logic allows for the use of specialised logics for specialized knowledge.

For example, in Defeasible Logic Programming (Garcia+Simari) and Extended Logic Programming (Prakken+Sartor) where Δ_1 is a set of strict rules and Δ_2 is a set of defeasible rules, and each of $\vdash_1$ and $\vdash_2$ are modus ponens).

Combined logics for argumentation

$(\Delta_1, \Delta_2) \vdash \alpha$ iff

$(\Delta_1, \Delta_2) \vdash \beta_1$ and $\ldots (\Delta_1, \Delta_2) \vdash \beta_n$

and $(\Delta_1 \cup \{\beta_1, \ldots, \beta_n\} \vdash_1 \alpha$ or $\Delta_2 \cup \{\beta_1, \ldots, \beta_n\} \vdash_2 \alpha)$

E.g. Δ_1 is strict and Δ_2 is defeasible.

$$\Delta_1 = \{a, a \rightarrow c, c \wedge b \rightarrow e\}$$

$$\Delta_2 = \{a \rightarrow b, e \rightarrow f\}$$

Hence

$$(\Delta_1, \Delta_2) \vdash f$$

Combined logics for argumentation

A disadvantage is that counter-intuitive reasoning may arise (as identified by Caminada+Amgoud) when using weak logics (e.g. logics without contraposition).

E.g. Δ_1 is strict and Δ_2 is defeasible.

$$\Delta_1 = \{weddingring, clubber, married \rightarrow spouse, single \rightarrow \neg spouse\}$$

$$\Delta_2 = \{weddingring \Rightarrow married, clubber \Rightarrow single\}$$

Hence

$$(\Delta_1, \Delta_2) \vdash married \quad (\Delta_1, \Delta_2) \vdash spouse$$

$$(\Delta_1, \Delta_2) \vdash single \quad (\Delta_1, \Delta_2) \vdash \neg spouse$$

Caminada+Amgoud suggest some interesting postulates for constraining argument systems (taking into account the definitions for entailment, attack and extensions).

Combined logics for argumentation

A more cautious approach to combining logics "ring-fences" the knowledgebase of one of the logics.

$$(\Delta_1, \Delta_2) \vdash \alpha \text{ iff } \Delta_1 \vdash_1 \alpha$$
$$\text{or } ((\Delta_1, \Delta_2) \vdash \beta_1 \text{ and } \ldots (\Delta_1, \Delta_2) \vdash \beta_n$$
$$\text{and } \Delta_2 \cup \{\beta_1, \ldots, \beta_n\} \vdash_2 \alpha)$$

E.g. Δ_1 is strict and Δ_2 is defeasible.

$$\Delta_1 = \{weddingring, clubber, married \rightarrow spouse, single \rightarrow \neg spouse\}$$
$$\Delta_2 = \{weddingring \rightarrow married, clubber \rightarrow single\}$$

Hence

$$(\Delta_1, \Delta_2) \vdash married \quad (\Delta_1, \Delta_2) \not\vdash spouse$$
$$(\Delta_1, \Delta_2) \vdash single \quad (\Delta_1, \Delta_2) \not\vdash \neg spouse$$

Ontology-based argumentation

- Δ_1 is a set of defeasible rules of the form $\beta_1 \wedge \ldots \beta_n \rightarrow \beta_{n+1}$ and $\vdash_1$ is modus ponens.
- Δ_2 is a set of classical or description logic formulae and $\vdash_2$ is classical/description logic consequence relation.

$(\Delta_1, \Delta_2) \vdash \alpha$ iff

$$(\Delta_1, \Delta_2) \vdash \beta_1 \text{ and } \ldots (\Delta_1, \Delta_2) \vdash \beta_n$$

$$\text{and } (\Delta_1 \cup \{\beta_1, \ldots, \beta_n\} \vdash_1 \alpha \text{ or } \Delta_2 \cup \{\beta_1, \ldots, \beta_n\} \vdash_2 \alpha)$$

E.g. $\Delta_1 = \{a \rightarrow b, e \rightarrow f\}$ and $\Delta_2 = \{a, \neg b \vee \neg c, \neg e \rightarrow c\}$.

$$(\Delta_1, \Delta_2) \vdash f$$

Ontology-based argumentation

Let Δ_1 be set of defeasible rules and let Δ_2 be an ontology.

An **argument** is a pair $\langle \Phi, \alpha \rangle$ such that

1. Φ is consistent
2. Φ is a minimal subset of Δ_1 s.t. $(\Phi, \Delta_2) \vdash \alpha$

Example: Let $\Delta_1 = \{b \wedge c \rightarrow d, d \rightarrow f, g \rightarrow \neg d, i \rightarrow \neg f\}$

and $\Delta_2 = \{a, a \rightarrow b, c, h, h \rightarrow g, i\}$

$\langle \{b \wedge c \rightarrow d, d \rightarrow f\}, f \rangle$

$\langle \{g \rightarrow \neg d\}, \neg d \rangle$ (undercut)

$\langle \{i \rightarrow \neg f\}, \neg f \rangle$ (rebut)

Ontology-based argumentation

Some of the content in the OWL ontology

CLASS NAMES

- DrugRegimes
  - Bisphosphonates
    - ZoledronicAcid
  - HormoneRA
    - Tamoxifen
      - Tamoxifen5Yr
      - Tamoxifen2Yr
    - Anastrazole
  - Diseases
  - Breast Cancer
    - ERPos Breast Cancer
    - ERNeg Breast Cancer
  - EndometrialCancer

PROPERTIES

hasDisease(Person, Disease)
hasTreatment(Person, Treatment)
hasAge(Person, int)

INSTANCES

Tamoxifen5Yr(TamCourseA)
Tamoxifen2Yr(TamCourseB)
Anastrazole(AnastrazoleCourseA)
ZoledronicAcid(ZolendronateA)

Example from Williams & Hunter, ICTAI'07

Ontology-based argumentation

An example of a defeasible rule implying changed risk of breast cancer disease-free survival.

Women(x)

$\wedge$ hasDisease(x,y) $\wedge$ EarlyBreastCancer(y) $\wedge$ ERPositiveDisease(y)

$\wedge$ hasTreatment(x,z) $\wedge$ Tamoxifen5Yr(z)

$\wedge$ IncreasedBrCaDFS(a)

$\Rightarrow$ hasDeltaRisk(x,a)

Example from Williams & Hunter, ICTAI'07

Ontology-based argumentation

- **Exploit the ontology as a repository of strict knowledge:** Use instances in ontology (ABox) as facts, and use class + relationship definitions in ontology (TBox) to infer further facts.
- **Exploit rigour of the ontology for the rule language:** Use class + relationship names of ontology as the predicate symbols in rules.
- **Exploit ontology reasoning software:** Evaluate each condition in a rule by querying the ontology.
- **Exploit argumentation system:** For generating and comparing arguments (e.g. DeLP, ASPIC,CASAPI, etc).
- **Exploit the ontology to determine conflict between pairs of atoms:** Conflict is redefined in terms of inconsitency in the ontology.

Conclusions

- Most logical argument systems are based on a simple defeasible logic, but there are many other base logic available in KR and CS.
- There are a number of ways that we can compare base logics
- Choice of base logic is important in designing a logical argument system
- It is useful to systematically combine base logics for applications (e.g. description logic with either defeasible logic or classical logic).