A characterization of collective conflict for defeasible argumentation

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Motivation

- To define a recursive semantics (Pollock, 2009) for warranted conclusions in a general framework (without levels of strength)
  - A collective (non-binary) notion of conflict between arguments
  - Direct and indirect consistency without distinguishing between direct and indirect conflicts between arguments

- To extend the recursive semantics to an argumentation framework with levels of preference (defeasibility) by providing a level-wise definition of warranted and blocked conclusions (Alsinet et al., 2008)

- To specialize the warrant recursive semantics for the particular framework of Defeasible Logic Programming (García and Simari, 2004), we refer to this formalism as RP-DeLP (Recursive Possibilistic Defeasible Logic Programming)

General defeasible argumentation framework

- Language based on a propositional logic $(\mathcal{L}, \vdash)$ with a special symbol $\perp$ for contradiction
- Knowledge base: (Π, Δ, Σ), where $\Pi, \Delta, \Sigma \subseteq \mathcal{L}$ and $\Pi \not\vdash \perp$
  - Π: strict knowledge (true formulas)
  - Δ: defeasible knowledge (formulas for which we have reasons to believe they are true)
  - Σ: Set of formulas over which arguments can be built
- An argument for a formula $\varphi \in \Sigma$ is is a pair $\mathcal{A} = \langle A, \varphi \rangle$, with $A \subseteq \Delta$ such that:
  1. $\Pi \cup A \not\vdash \perp$, and
  2. A is minimal (w.r.t. set inclusion) such that $\Pi \cup A \vdash \varphi$.
- Defeasible argument: $\langle A, \varphi \rangle$ such that $A \neq \emptyset$
- A formula $\varphi \in \Sigma$ is justifiable if there exists $A \subseteq \Delta$ such that $\langle A, \varphi \rangle$ is an argument

General defeasible argumentation framework

- The notion of subargument is referred to defeasible arguments and expresses an incremental notion of proof between arguments

Example: $(\Pi, \Delta, \Sigma) = (\{r\}, \{r \rightarrow p \wedge q\}, \{p, q, p \wedge q\})$

- $\mathcal{A}_1 = \langle A, p \rangle$, $\mathcal{A}_2 = \langle A, q \rangle$ and $\mathcal{A}_3 = \langle A, p \wedge q \rangle$ are arguments for different formulas with support $A = \{r \rightarrow p \wedge q\}$
- Subargument relation: $\mathcal{A}_3 \sqsubset \mathcal{A}_1$ and $\mathcal{A}_3 \sqsubset \mathcal{A}_2$ since $p \wedge q \vdash p$ and $p \wedge q \vdash q$
- No subargument relation: $\mathcal{A}_1 \not\sqsubset \mathcal{A}_3$ and $\mathcal{A}_2 \not\sqsubset \mathcal{A}_3$ since $p \nvdash p \wedge q$ and $q \nvdash p \wedge q$

Collective conflict: motivation

Aim: To find a set of justifiable formulas consistent w.r.t. the strict knowledge

Example: $\mathcal{P} = (\Pi, \Delta, \Sigma)$ with

$$\Pi = \{a \wedge b \rightarrow \neg p\},\ \Delta = \{a, b, p\} \text{ and } \Sigma = \{a, b, p, \neg p\}.$$

- $\langle\{p\}, p\rangle$, $\langle\{b\}, b\rangle$, $\langle\{a\}, a\rangle$ are arguments that justify p, b and a, respectively.
- No binary conflict relation: $\Pi \cup \{a, b\} \nvdash \bot$, $\Pi \cup \{a, p\} \nvdash \bot$ and $\Pi \cup \{b, p\} \nvdash \bot$
- Collective conflict relation: $\Pi \cup \{a, b, p\} \vdash \bot$
- a, b and p are (collectively) inconsistent w.r.t. Π

Warrant semantics: general framework

Input: a KB $\mathcal{P} = (\Pi, \Delta, \Sigma)$

Output: a pair $(\mathit{Warr}, \mathit{Block})$ such that

- An argument $\langle A, \varphi \rangle$ is either warranted or blocked whenever each subargument is warranted; then, it is eventually warranted if φ is not involved in any conflict, otherwise it is blocked
- $\Pi \cup \mathit{Warr} \nvdash \perp$
- $\Pi \cup \mathit{Block} \vdash \perp$
- $\mathit{Warr} = \textit{s-Warr} \cup \textit{d-Warr}$ with
  - $\textit{s-Warr} = \{\varphi \mid \Pi \vdash \varphi\} \cap \Sigma$
  - $\textit{d-Warr}$ and Block are required to satisfy the following recursive constraints

Warrant semantics: general framework

Input: a KB $\mathcal{P} = (\Pi, \Delta, \Sigma)$

Output: a pair $(Warr, Block)$ where $Warr = s\text{-}Warr \cup d\text{-}Warr$

- Recursive definition of $d\text{-}Warr$ and $Block$:

- A defeasible argument $\langle A, \varphi \rangle$ is valid (not rejected) if all subarguments are warranted

- For every valid argument $\langle A, \varphi \rangle$

  - $\varphi \in d\text{-}Warr$ whenever there does not exist a set of valid arguments G such that

    (i) Arguments in G do not depend on $\langle A, \varphi \rangle$

    (ii) G and $\langle A, \varphi \rangle$ are inconsistent w.r.t. Π

  - otherwise, $\varphi \in Block$

Warrant semantics: example

$\mathcal{P} = (\Pi, \Delta, \Sigma)$, with $\Pi = \{a \rightarrow y, b \wedge c \rightarrow \neg y\}$, $\Delta = \{a, b, c, \neg c\}$ and $\Sigma = \{a, b, c, \neg c, y, \neg y\}$

- $s\text{-}Warr = \emptyset$
- Valid arguments: $\langle \{a\}, a \rangle$, $\langle \{b\}, b \rangle$, $\langle \{c\}, c \rangle$ and $\langle \{\neg c\}, \neg c \rangle$
- Each valid argument is either warranted or blocked:

  $\Pi \cup \{a, b, c\} \vdash \bot \quad \Rightarrow \quad a,\ b$ and c are blocked conclusions

  $\Pi \cup \{c, \neg c\} \vdash \bot \quad \Rightarrow \quad c$ and $\neg c$ are blocked conclusions
- Output: $Warr = \emptyset$ and $Block = \{a, b, c, \neg c\}$
- Intuition: every conclusion in *Block* is (individually) valid, however all together are contradictory w.r.t. Π
- Remark: $y, \neg y \notin Block$ since arguments $\langle \{a\}, y \rangle$ and $\langle \{b, c\}, \neg y \rangle$ depend on a, b and $c \notin Warr$

Warrant semantics: closure property (general framework)

Let $(Warr, Block)$ be an output for $\mathcal{P} = (\Pi, \Delta, \Sigma)$

If $\Pi \cup Warr \vdash \varphi$ then $\varphi \in Warr$ whenever

1. $\varphi \in \Sigma$
2. there exits a valid argument for φ

Example: $\mathcal{P} = (\Pi, \Delta, \Sigma)$ with $\Pi = \{a \wedge b \rightarrow y\}$,
$\Delta = \{s \rightarrow a, \sim s \rightarrow b, s, \sim s\}$ and $\Sigma = \{a, b, y\}$

Output: $Warr = \{a, b\}$ and $Block = \emptyset$

Problem: $\Pi \cup Warr \vdash y$ and $y \in \Sigma$, however $y \notin Warr$

Intuition: there does not exist a valid argument for y since the proof of y is based on a and b which are respectively based on s and $\sim s$

Solution: to extend $\Sigma = \{a, b, y, s, \sim s\} \Rightarrow$
$Warr = \emptyset$ and $Block = \{s, \sim s\}$

Introducing a preference ordering on arguments

- Stratified knowledge base: $(\Pi, \Delta, \preceq, \Sigma)$ such that $\preceq$ is a total pre-order on the set of defeasible formulas Δ representable by a necessity measure $N : \mathcal{L} \rightarrow [0, 1]$

$$\varphi \preceq \psi \text{ iff } N(\varphi) \leq N(\psi)$$

1. $N(\top) = 1, N(\bot) = 0,$
2. $N(\varphi \wedge \psi) = \min(N(\varphi), N(\psi))$
3. $N(\varphi) = 1 \text{ iff } \Pi \vdash \varphi$

- Strength of an argument:

$$s(\langle A, \varphi \rangle) = 1 \text{ if } A = \emptyset, \text{ and}$$
$$s(\langle A, \varphi \rangle) = \min\{N(\psi) \mid \psi \in A\}, \text{ otherwise.}$$

Warrant semantics: levels of strength

Input: $(\Pi, \Delta, \preceq, \Sigma)$ such that $1 > \alpha_1 > \ldots > \alpha_p \geq 0$ are the strengths of defeasible arguments

Output: a pair $(Warr, Block)$ such that

- $Warr = s\text{-}Warr \cup d\text{-}Warr$
- $s\text{-}Warr = \{\varphi \mid \Pi \vdash \varphi\} \cap \Sigma$
- $d\text{-}Warr = \{d\text{-}Warr(\alpha_1), \ldots, d\text{-}Warr(\alpha_p)\}$, where $d\text{-}Warr(\alpha_i)$ is the set of warranted conclusions with strength α_i
- $Block = \{Block(\alpha_1), \ldots, Block(\alpha_p)\}$, where $Block(\alpha_i)$ is the set of blocked conclusions with strength α_i

Extension: a level-wise construction of the sets of conclusions $Warr(\alpha_i)$ and $Block(\alpha_i)$

Warrant semantics: levels of strength

Recursive definition of $Warr(\alpha_i)$ and $Block(\alpha_i)$:

- A defeasible argument $\langle A, \varphi \rangle$ of strength α_i is valid (not rejected) if
  1. all subarguments are warranted
  2. $\langle A, \varphi \rangle$ is consistent w.r.t. Π and $\cup_{\beta > \alpha_i} d\text{-}Warr(\beta)$
  3. $\varphi \notin \cup_{\beta > \alpha_i} d\text{-}Warr(\beta)$
  4. $\varphi \notin \cup_{\beta > \alpha_i} Block(\beta)$
  5. $\{\varphi, \psi\} \not\vdash \bot$ for all $\psi \in \cup_{\beta > \alpha_i} Block(\beta)$
- For every valid argument $\langle A, \varphi \rangle$ of strength α_i
  - $\varphi \in d\text{-}Warr(\alpha_i)$ whenever there does not exist a set G of valid arguments of strength α_i such that
    - (i) Arguments in G do not depend on $\langle A, \varphi \rangle$
    - (ii) G and $\langle A, \varphi \rangle$ are inconsistent w.r.t. Π and $\cup_{\beta > \alpha_i} d\text{-}Warr(\beta)$
  - otherwise, $\varphi \in Block(\alpha_i)$

Levels of strength: example

$\Pi = \{a \wedge b \rightarrow \neg p\}$, $\Delta = \{a, b, p\}$ and $\Sigma = \{a, b, p, \neg p\}$ extended with two levels of defeasibility:

$$\begin{array}{ccccc} \{a, b\} & \prec & p & & \\ \alpha_2 & < & \alpha_1 & < & 1 \end{array}$$

- $s\text{-}Warr = \emptyset$
- Level α_1: one valid argument $\langle \{p\}, p \rangle$

$$d\text{-}Warr(\alpha_1) = \{p\} \text{ and } Block(\alpha_1) = \emptyset$$

- Level α_2:
  - Two valid arguments: $\langle \{a\}, a \rangle$ and $\langle \{b\}, b \rangle$
  - $\Pi \cup d\text{-}Warr(\alpha_1) \cup \{a, b\} \vdash \perp \Rightarrow$ a and b are blocked
  - $a, b \notin d\text{-}Warr(\alpha_2) \Rightarrow \langle \{a, b\}, \neg p \rangle$ is not valid $\Rightarrow \neg p$ is rejected

$$d\text{-}Warr(\alpha_2) = \emptyset \text{ and } Block(\alpha_2) = \{a, b\}$$

- Output: $Warr = \{p\}$ and $Block = \{a, b\}$

Levels of strength: example

$\Pi = \{a \rightarrow y, b \wedge c \rightarrow \neg y\}$, $\Delta = \{a, b, c, \neg c\}$ and $\Sigma = \{a, b, c, \neg c, y, \neg y\}$ extended with three levels of defeasibility:

$$\begin{array}{ccccccc} \neg c & \prec & c & \prec & \{a, b\} & & \\ \alpha_3 & < & \alpha_2 & < & \alpha_1 & < & 1 \end{array}$$

- $s\text{-}Warr = \emptyset$
- Level α_1: we have not only a, b and y with valid arguments not generating conflict, but also $\langle\{a, b\}, \neg c\rangle$

$$d\text{-}Warr(\alpha_1) = \{a, b, y, \neg c\} \text{ and } Block(\alpha_1) = \emptyset$$

- Level α_2:
  - $\Pi \cup d\text{-}Warr(\alpha_1) \cup \{c\} \vdash \bot \Rightarrow c$ is rejected
  - $\langle\{b, c\}, \neg y\rangle$ is based on $c \notin d\text{-}Warr(\alpha_1) \Rightarrow y$ is rejected

$$d\text{-}Warr(\alpha_2) = \emptyset \text{ and } Block(\alpha_2) = \emptyset$$

- Level α_3: $\neg c \in d\text{-}Warr(\alpha_1) \Rightarrow$

$$d\text{-}Warr(\alpha_3) = \emptyset \text{ and } Block(\alpha_3) = \emptyset$$

A particular case: Recursive Possibilistic Defeasible Logic Programming (RP-DeLP)

RP-DeLP program: $(\Pi, \Delta, \preceq, \Sigma)$ over the logic $(\mathcal{L}_R, \vdash_R)$, where

- $\mathcal{L}_R$ consist of atoms $p, q, \ldots$ extended with a (negated) atom "$\sim p$" for each original atom p
- Formulas of $\mathcal{L}_R$ are of the form $Q \leftarrow P_1 \wedge \ldots \wedge P_k$, where $Q, P_1, \ldots, P_k$ are literals (atoms p and $\sim p$)
- $\vdash_R$ is defined by instances of the modus ponens rule of the form: $\{Q \leftarrow P_1 \wedge \ldots \wedge P_k, P_1, \ldots, P_k\} \vdash_R Q$
- Σ consists of the set of all literals of $\mathcal{L}_R$

Indirect consistency and closure w.r.t. the strict knowledge: Let $(Warr, Block)$ be an output for a RP-DeLP program

- $\Pi \cup Warr \not\vdash_R \bot$
- If $\Pi \cup Warr \vdash_R Q$ then $Q \in Warr$

RP-DeLP programs with unique output

- Circular definitions of warranty that emerge when considering conflicts among arguments $\Rightarrow$ The output $(Warr, Block)$ for a RP-DeLP program is not unique

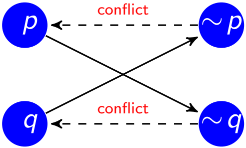
- Circular definitions of warranty are characterized by means of warrant dependency graphs of RP-DeLP programs

- Warrant dependency graph: represents conflict and support dependences among arguments w.r.t. the strict knowledge and a set of warranted conclusions

- A pair $(Warr, Block)$ is the unique output for a RP-DeLP program iff for all literal $L \in Warr$ there is no cycle in the warrant dependency graph for L w.r.t. $Warr$

Example: an empty set of strict clauses and one defeasibility level α_1 for $\Delta = \{p, q, \sim p \leftarrow q, \sim q \leftarrow p\}$

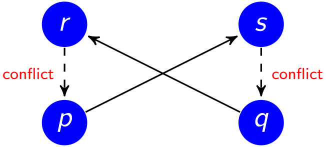
- $s\text{-}Warr = \emptyset$
- Level α_1:
  - Two valid arguments: $\langle \{p\}, p \rangle$ and $\langle \{q\}, q \rangle$
  - Two (possible) valid arguments (support relations with p and q): $\langle \{q, \sim p \leftarrow q\}, \sim p \rangle$ and $\langle \{p, \sim q \leftarrow p\}, \sim q \rangle$
  - Two (possible) conflicts at level α_1:
    $\{p, \sim p\} \vdash_R \bot$ and $\{q, \sim q\} \vdash_R \bot$
  - $\Rightarrow$ A cycle at the warrant dependency graph:



- Two outputs:
  - $Output_1 = (Warr_1, Block_1)$ with
    $Warr_1 = \{p\}$ and $Block_1 = \{q, \sim q\}$
  - $Output_2 = (Warr_2, Block_2)$ with
    $Warr_2 = \{q\}$ and $Block_2 = \{p, \sim p\}$

Example: $\mathcal{P} = (\Pi, \Delta)$, with $\Pi = \{y, \sim y \leftarrow p \wedge r, \sim y \leftarrow q \wedge s\}$ and one defeasibility level α_1 for $\Delta = \{p, q, r \leftarrow q, s \leftarrow p\}$

- $s\text{-}Warr = \{y\}$
- Level α_1:
  - Two valid arguments: $\langle\{p\}, p\rangle$ and $\langle\{q\}, q\rangle$
  - Two (possible) valid arguments (support relations with p and q): $\langle\{q, r \leftarrow q\}, r\rangle$ and $\langle\{p, s \leftarrow p\}, s\rangle$
  - Two (possible) conflicts at level α_1:
  $\Pi \cup s\text{-}Warr \cup \{p, r\} \vdash_R \perp$ and $\Pi \cup s\text{-}Warr \cup \{q, s\} \vdash_R \perp$
  - $\Rightarrow$ A cycle at the warrant dependency graph:



- Two outputs:
  - $Output_1 = (Warr_1, Block_1)$ with
    $Warr_1 = \{p\}$ and $Block_1 = \{q, s\}$
  - $Output_2 = (Warr_2, Block_2)$ with
    $Warr_2 = \{q\}$ and $Block_2 = \{p, r\}$

Conclusions

- A recursive semantics for determining the warranty status of arguments in defeasible argumentation
  - A general defeasible argumentation framework based on a propositional logic
  - Levels of defeasibility $\Rightarrow$ Strength of arguments
  - RP-DeLP
- A collective notion of conflict
- Direct and indirect consistency w.r.t. the strict knowledge
- Circular definition of warranty $\Rightarrow$ Warrant dependency graph of a RP-DeLP program
- Not based on dialectical trees $\Rightarrow$ It is not necessary to explicitly compute all the possible arguments for a given literal to check whether it is warranted
- RP-DeLP programs with unique output: implementation with a worst-case complexity in P^{NP}