# Tractability in Value-based Argumentation

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# Overview

- Value-based argumentation (VAF).
- Problems in VAFs – SBA & OBA
- Value-graphs and “relevant” audiences.
- Polynomial time decidable cases.
- Summary

# Value-Based Frameworks I

- Extends $<X,A>$ in AFs to $<X,A,V,\eta>$
- $V=\{v_1,\ldots,v_k\}$ finite set of *values*
- $\eta : X \rightarrow V$ maps arguments to values.
- VAF semantics defined w.r.t. *audiences*

$\alpha$ – ordering of $V$

$v_i >_\alpha v_j$ – the *value* $v_i$ is ranked above the *value* $v_j$ by the audience $\alpha$

# Value-Based Frameworks II

- One consistency condition: every cycle in $<X,A>$ contains at least 2 distinctly valued arguments.
- Attacks succeed relative to *audiences.*
- Concept of "successful attack" by $x$ on $y$ becomes: $<x,y> \in A$ *and it is not the case that* $\eta(y) >_{\alpha} \eta(x)$

# Decision Problems in VAFs I

- Two main questions are
  SBA (Subjective Acceptance)
  OBA (Objective Acceptance)
- These ask if an argument is in the grounded extension of the AF induced by *at least one*/*every* audience (value ordering).
- i.e., the (provably acyclic) AFs formed by removing failing attacks: those for which the attacker's value is ranked lower than that of its target argument.

# Decision Problems in VAFs II

- Letting $k$ denote the number of values, simply *enumerating all possible audiences* gives algorithms for both problems that complete in $O(k!|X|)$ steps: form the reduced AF and test if $x$ is in its grounded extension.
- If $k$ is "*large*" ($k \geq 20$, say) this approach is, however, *impractical*.

# SBA & OBA Complexity

- In general SBA is NP-complete and OBA coNP-complete.
- These remain unchanged even when <X,A> is a *binary tree in which no value occurs more than three times*.
- Qn1: are there cases allowing SBA and OBA to be decided in *polynomial* (in |X| & k) *time* ?
- Qn2: to what extent can the O(k!|X|) bound be *improved generally*?

# Results

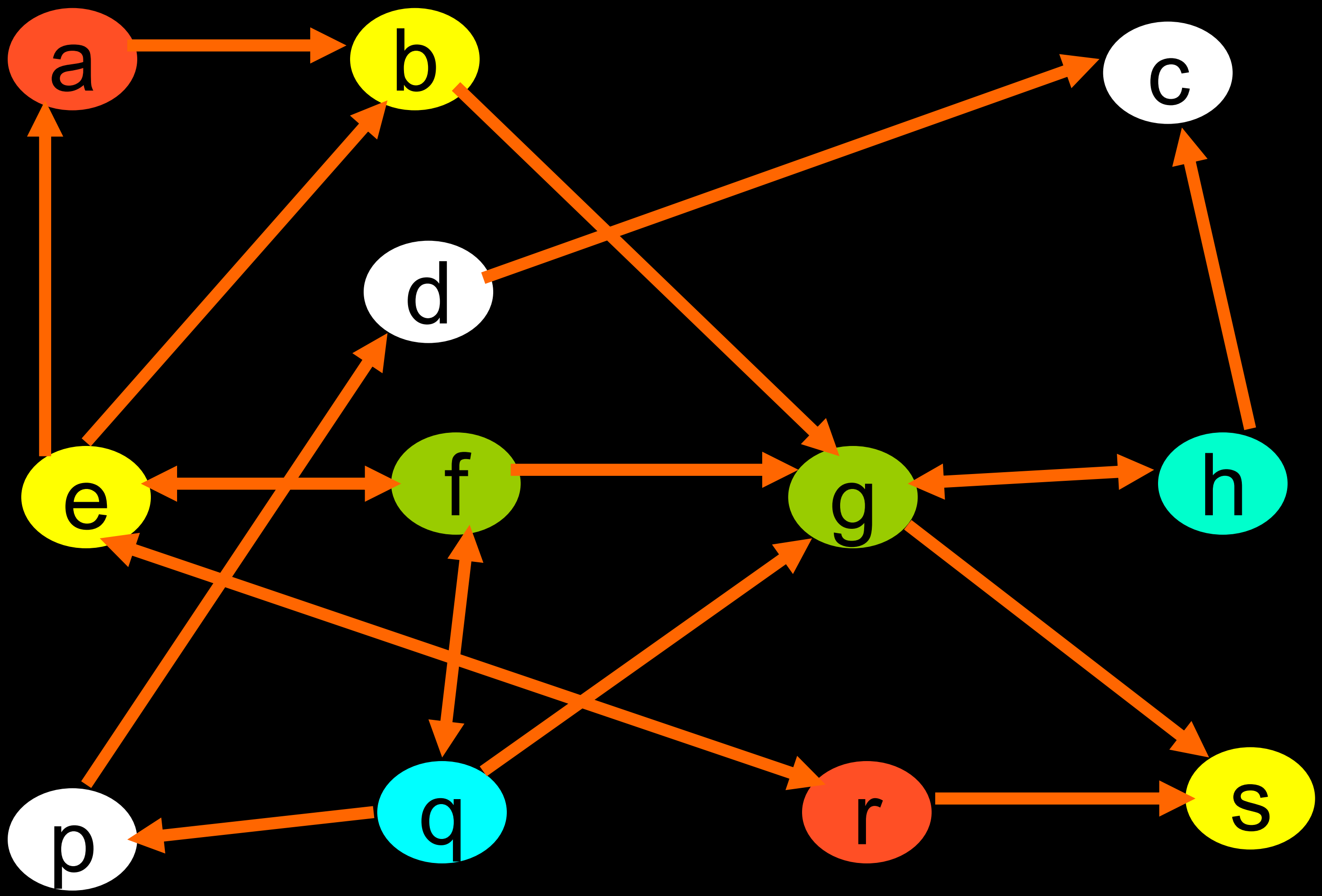
- A general class of conditions is described which –

a. In specific cases yield polynomial (in $|X|$ & $k$) time algorithms.

b. At worst have run-time $O(2^k|X|)$: significantly improving on $k!|X|$, noting that $\log_2 k! \approx O(k \log k)$.

# Value Graphs

- Given a VAF – $<X,A,V,\eta>$ – its *value graph* is the *digraph* $<V,B>$ in which
  $B = \{<i,j> : <x,y> \in A$ for some $x, y$ with $\eta(x)=i,$
  $\eta(y)=j \ \& \ i \neq j\}.$
- We concentrate on VAFs whose (undirected) value graph is a “*tree*”.
- Note that a value graph may be in this class even when the VAF yielding it is not.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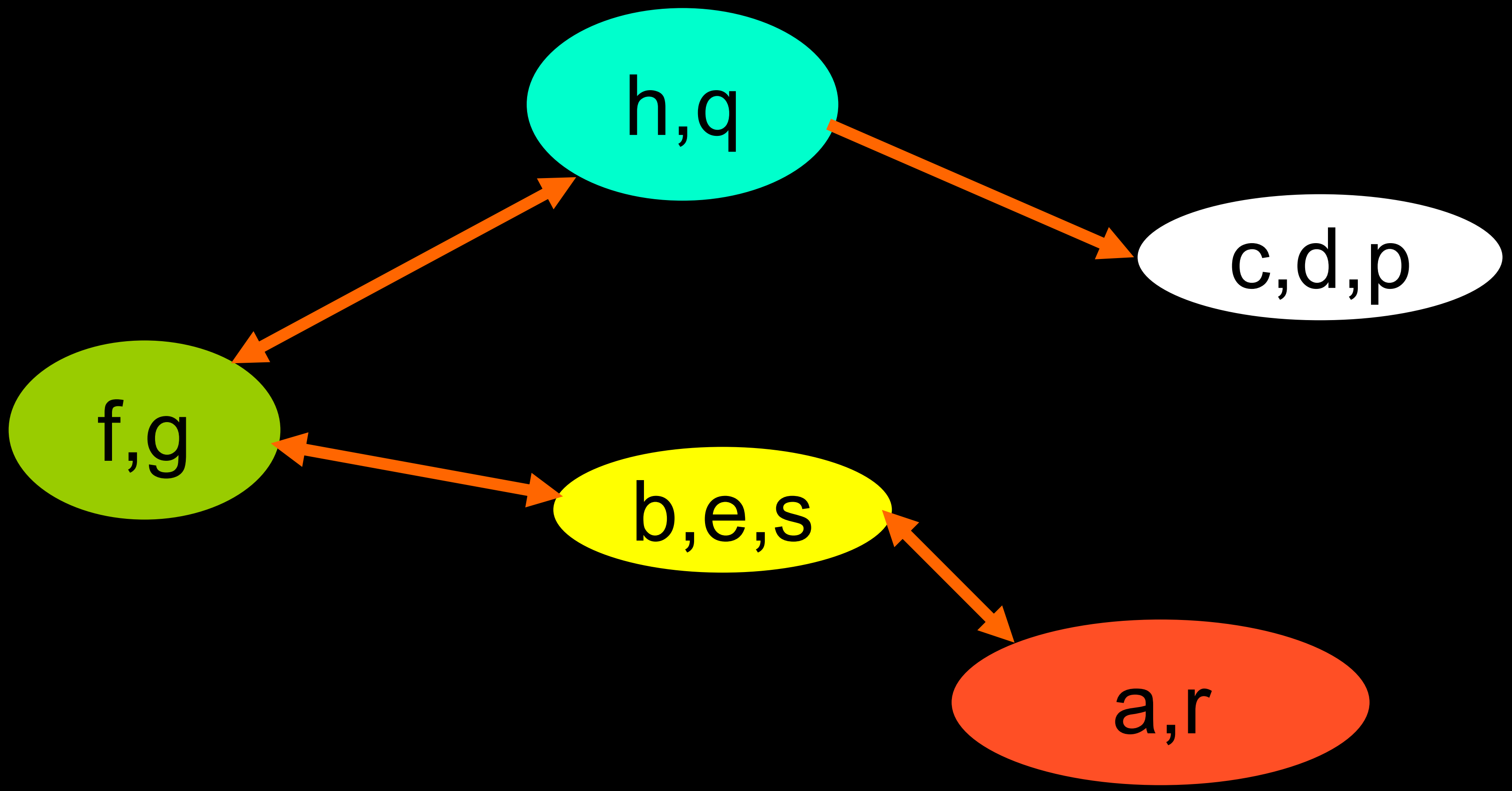
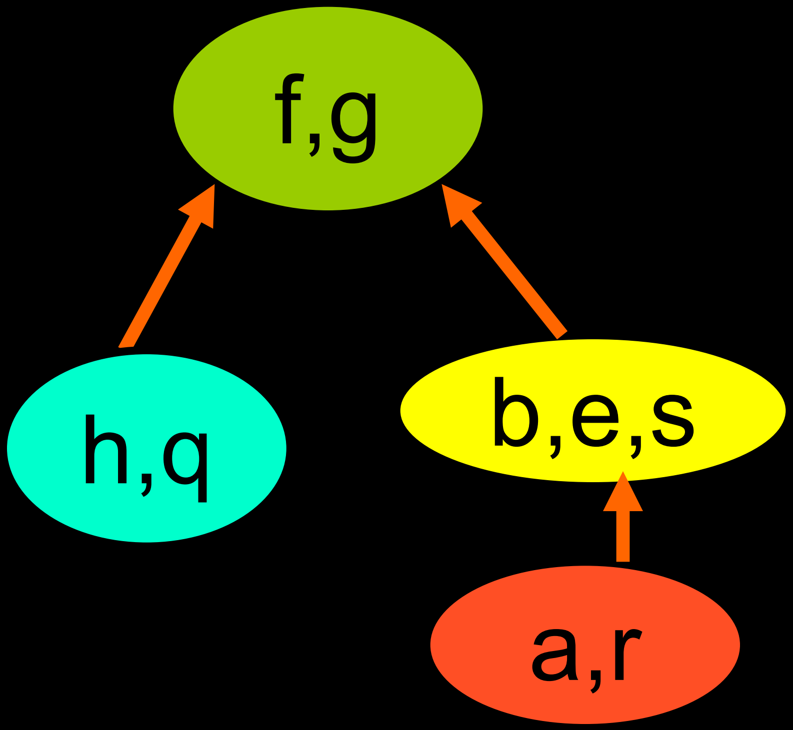

h,q
c,d,p
f,g
b,e,s
a,r

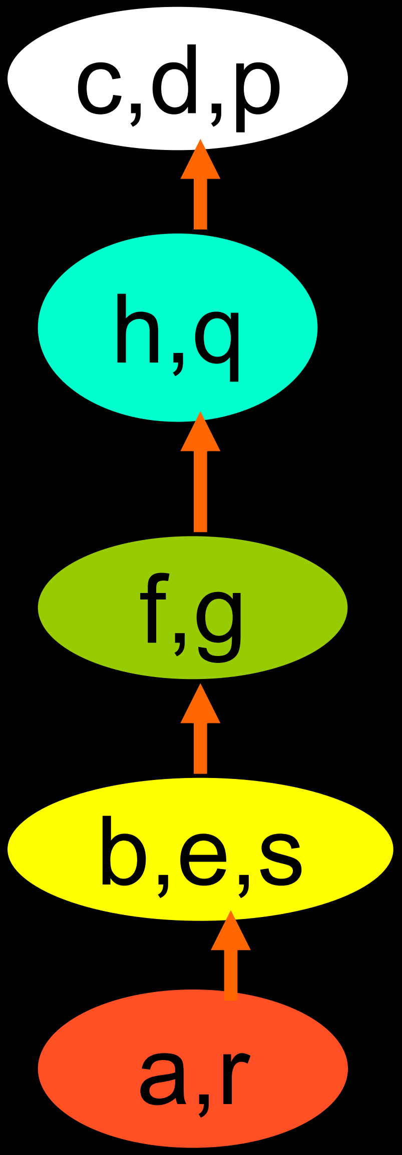
# Relevant Audiences I

- The example uses 5 values giving 120 specific audiences.
- Consider, e.g. the arguments {h,q}: these only "interact" with *white* values ({c,d,p}) and *green* ({f,g}); not with *red* ({a,r}) or *yellow* ({b,e,s}). In additon *white* values are *only attacked*.
- In total, only *partial* audiences need be considered in finding the status of h or q.

# Relevant Audiences II

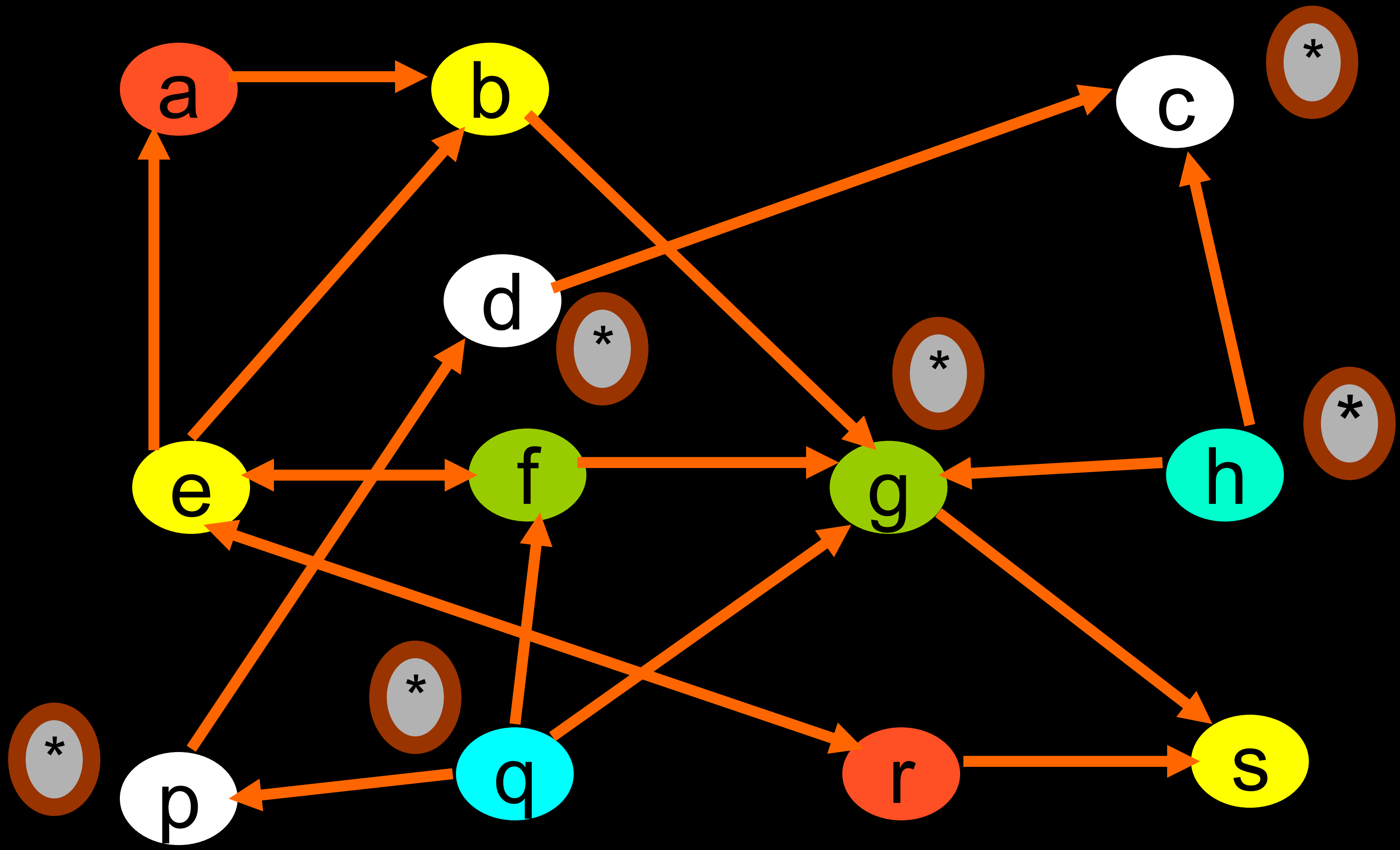
- For value graphs which are trees the notion of relevant audience is defined w.r.t. each value.
- For a value, v, orientate the tree to have root v with every edge directed "towards" the root.
- The relevant audiences w.r.t. v, informally are the partial audiences defining "connected" sub-trees of the value graph containing v.

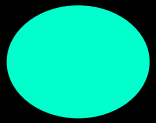
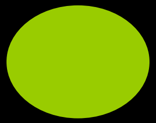
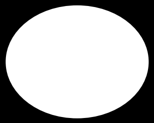


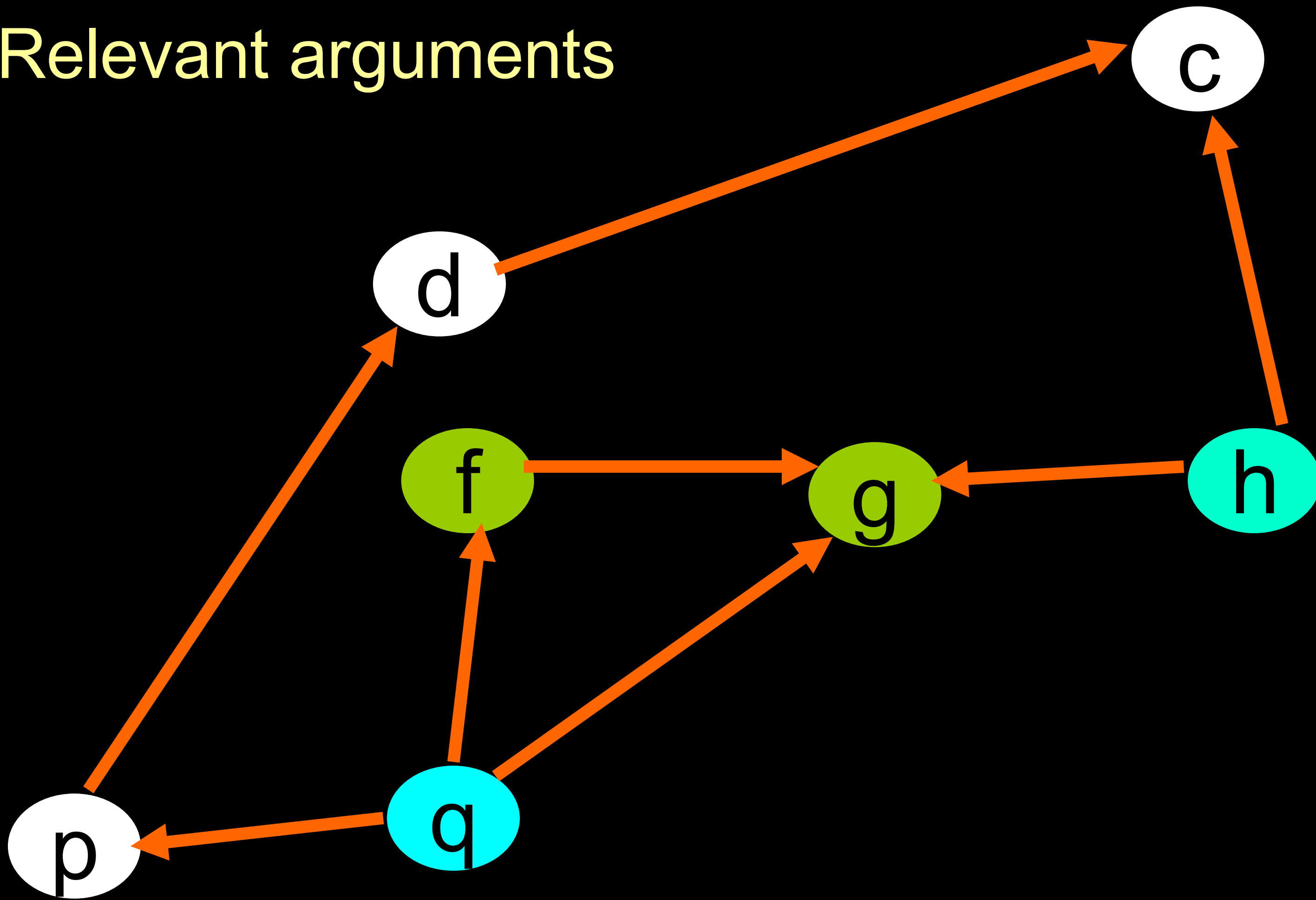

Relevant audiences induce connected subtrees

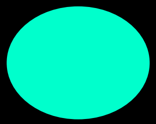
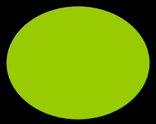
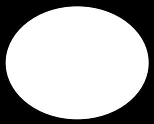
Leading to acyclic AFs from VAF

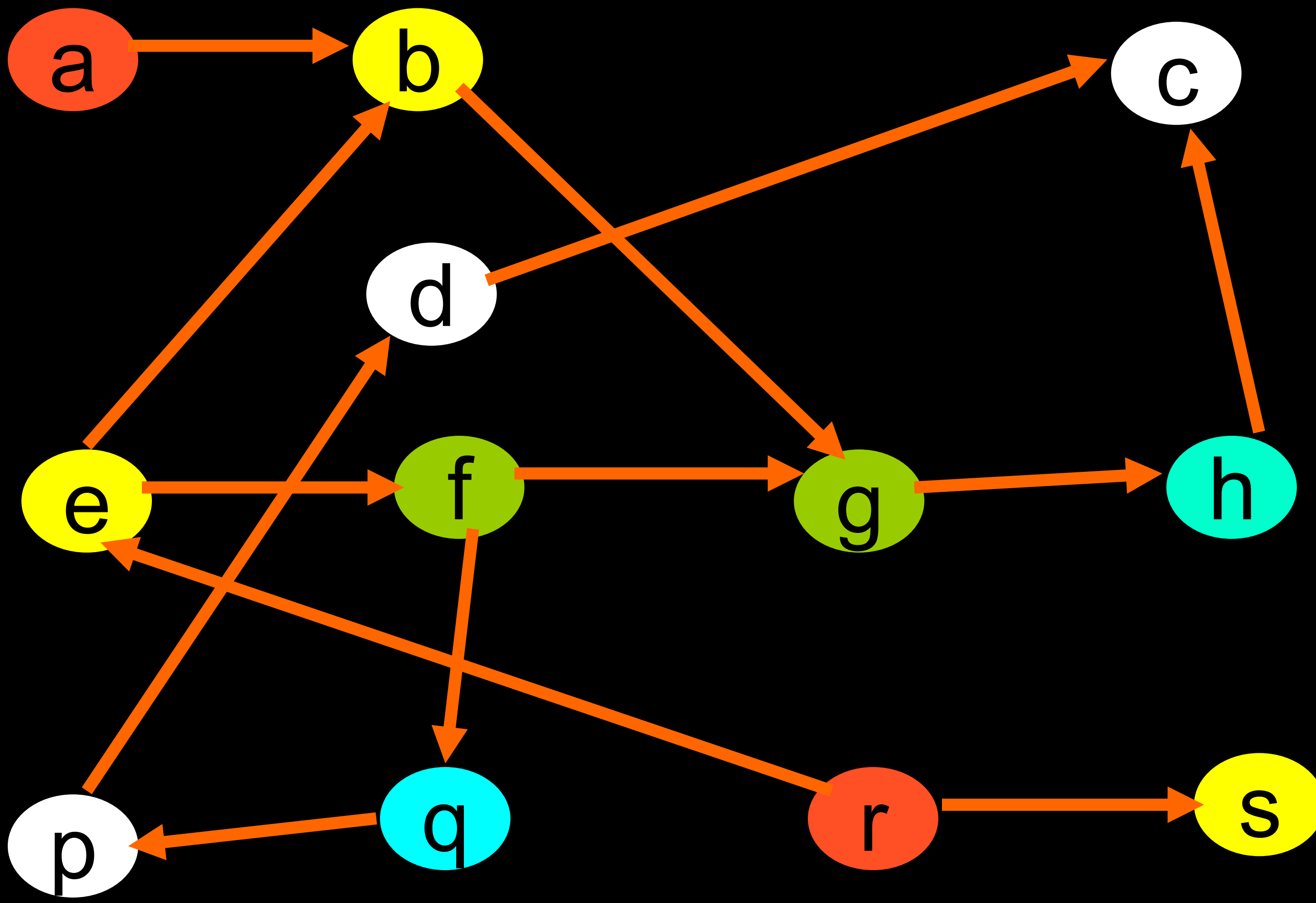



VAF induced by  > {   }

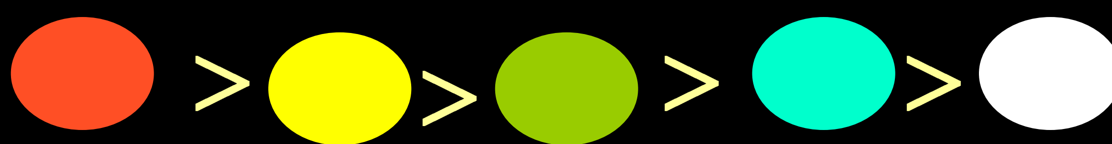
Relevant arguments




VAF induced by  > {   }

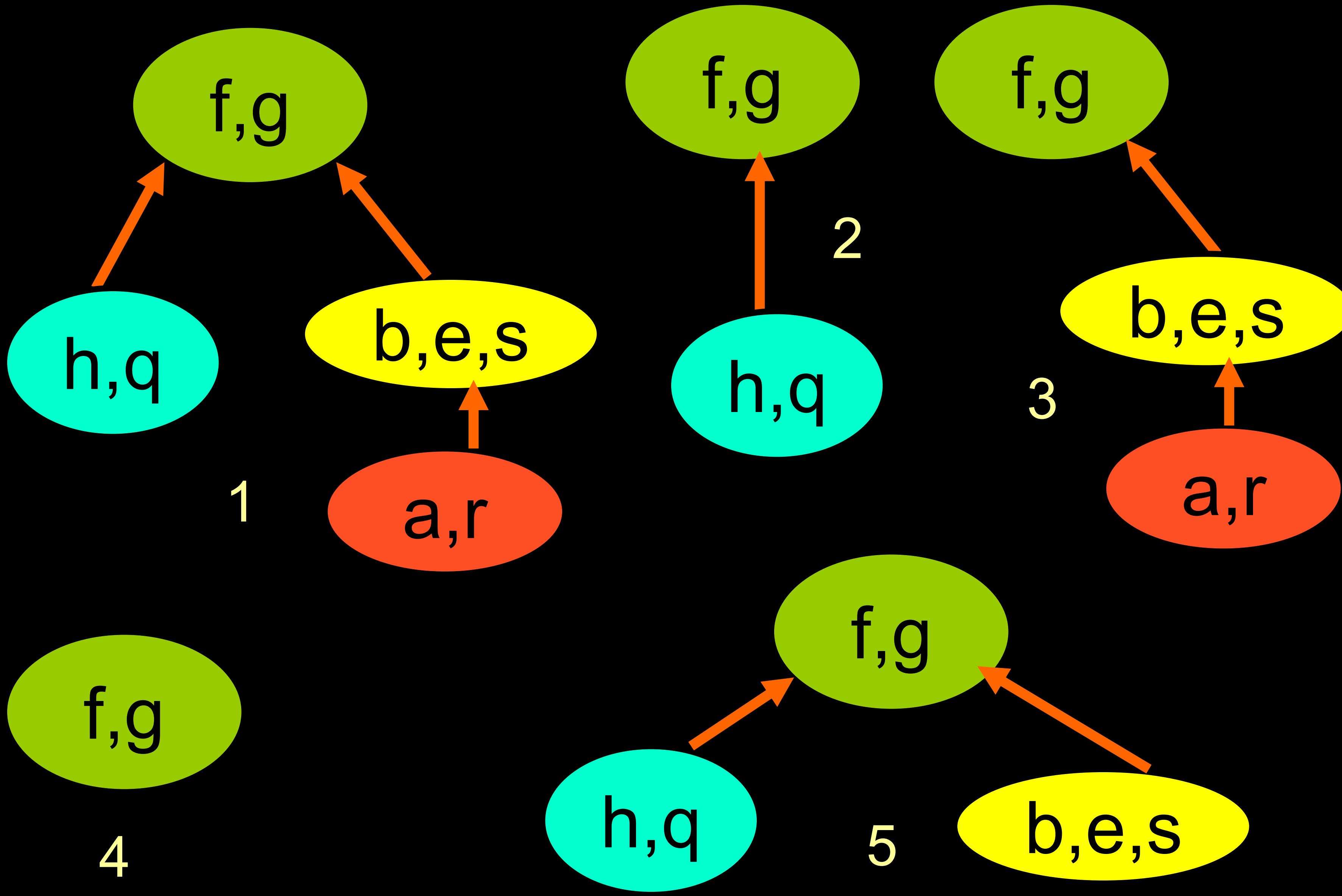



VAF induced by



# Summary of Properties

- To determine status of argument with value v, *only* those partial audiences inducing a *connected tree* with root v are significant.
- Each induces an *acyclic sub-graph* of the original VAF.
- The number of such partial audiences is at most $2^{k-1}$ and can be *much less*.


f,g
h,q
b,e,s
a,r
1
f,g
h,q
2
f,g
b,e,s
a,r
3
f,g
4
f,g
h,q
b,e,s
5

# Polynomial upper bounds I

- To decide SBA/OBA – enumerate all relevant audiences; form induced AF; test if argument of interest is accepted.
- The significant factor is the *total number* of such audiences.
- This in turn is determined by – the *number* of values with *more than one attack* in the tree; the *maximum* number of attacks on a value.

# Polynomial upper bounds II

- Let t=#values with >1 attack; d=max. #attacks on any value; k=#values.
- Then the number of relevant audiences is at most

$$\frac{(t(d-2)+k+1)^{td}}{(k-t)^{t-1} \times (t(d-1)+1)^{t(d-1)+1}}$$

This is polynomial (in k) when $t \times d = O(1)$

# Summary I

- In best case (value graph is a "*chain*") the approach gives $O(k^2)$ bound on number of audiences.
- The construction, however, *does not* extend to "*near tree*" structures.
- In particular, if the value graph has *treewidth* 2, SBA remains NP-complete, even when the treewidth decomposition is a *chain*.

# Summary II

- Concept of “relevant audience” extends to more general forms of value graph, i.e. *not just trees*.
- In particular, current work examines ideas of “audience covers” in value graphs and gives

  $$(1+e)((k-1)!)$$

  as an *upper bound* on the *maximum* number of audiences that *ever need to be considered* (i.e. $k!$ is not optimal).
- The extent to which further restrictions lead to improved methods is also a topic of current work.