Computation with Varied-Strength Attacks in Abstract Argumentation Frameworks

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Overview

- AFs with varied strength attacks. (AFV)
- Scenarios, updates & equilibria.
- Decision questions specific to AFVs.
- Complexity results.
- Conclusions & development.

AFs & varied-strength attacks (AFVs)

- Modifies standard <X,A> structure by –
  Partitioning A into $\{A_1, A_2, \ldots, A_n\}$
  Adding binary relation R over [n] to obtain 4 relations
- These describe comparative "*strengths*"
  j<<k – A_j is "*weaker*" than A_k (only $\langle k,j\rangle \in R$)
  j>>k – A_j is "*stronger*" than A_k (only $\langle j,k\rangle \in R$)
  j≈k – A_j "*equivalent force*" to A_k ($\langle j,k\rangle$ & $\langle k,j\rangle \in R$)
  j?k – A_j "*unknown difference*" from A_k (neither in R)

What does this buy?

- Many variants of AFs are concerned with rationales for *excluding attacks*. This is the case with PAFs, VAFs, EAFs, weighted systems, AFRAs, etc.
- AFV semantics *do not deal* with discounting attacks but are directed at distinguishing different "*qualities*" of *defences*.
- Informally, if x is "attacked" by y and we have a choice of p or q as counterattacks ("*defences*") then we prefer to use whichever of {<p,y>,<q,y>} is "*stronger*".
- and, of course, want such defences to be "*at least as*" "*powerful*" as the attack <y,x>.

Admissible Scenarios

- Let $S \subseteq X$ and $P \subseteq \{<<, >>, \approx, ?\}$. Structures of interest in AFVs are pairs $[S,P]$ called *attack scenarios*.
- Given $x \in X$, x is acceptable wrt $[S,P]$ if:
  $\forall y$ s.t. $<y,x> \in A_j$ $\exists z \in S$, $\rho \in P$ $<z,y> \in A_k$ & $k \rho j$
- $[S,P]$ is an *admissible scenario* if S is conflict-free & every $x \in S$ is acceptable wrt $[S,P]$.

Differentiating defences

- Suppose $\{u,x,y,z\}\subseteq X$ with $\langle u,x\rangle \in A_i$, $\langle y,u\rangle \in A_j$, $\langle z,u\rangle \in A_k$. Then:

  y *dominates* z (for x) if $k \gg j$.

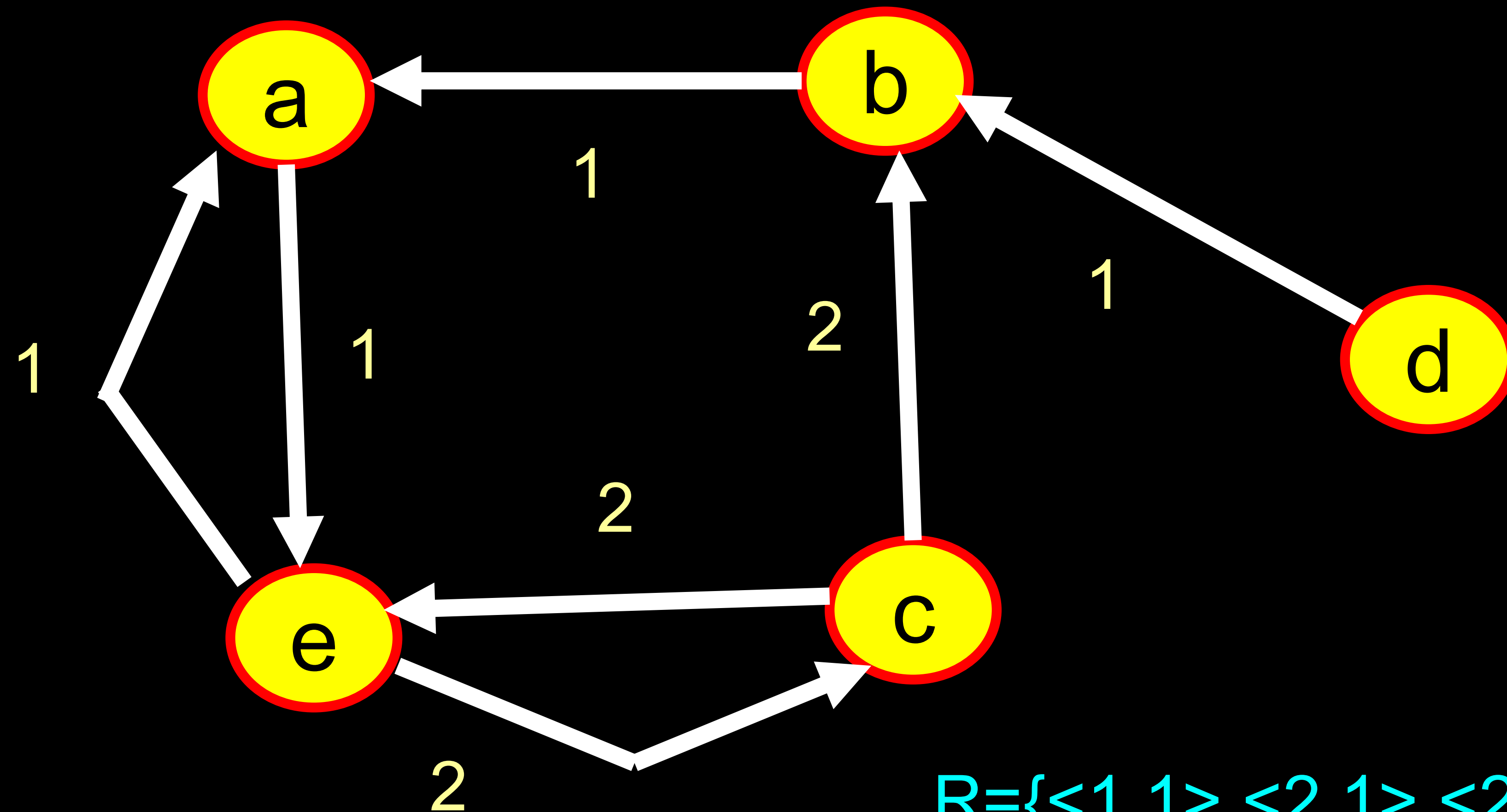
- [S,P] is a *stronger collective defence* (of x) than [T,Q] if –

  x is acceptable to [S,P] & [T,Q]

  $\forall y \in T$, $z \in S$ y does *not* dominate z (for x)

  $\exists p \in S$, $q \in T \setminus S$: p dominates q (for x) [{a,c},{>>,≈}]

stronger collective defence (for a) than [{a,d},{>>,≈}]



$R=\{\langle 1,1\rangle,\langle 2,1\rangle,\langle 2,2\rangle\}$

Top admissibility & upgrades

- The notion of "*top admissible scenario*" captures that of "*best defence*".
- [S,P] is *top admissible* if it is both admissible and for each $x \in S$ no admissible scenario [T,Q] with $x \in T$ provides a stronger collective defence (for x).
- If [T,P] is a stronger defence (for x) the *upgraded defence* – upgd(x,S,T,P) – is

  $[S \setminus def(x,[S,P]) \cup def(x,[T,P],P]$.
- Here, def(x,[S,P]) denotes the defenders of x in S.
- Upgraded defence: not *always admissible scenario*.
- [S,P] is *equilibrated* if no $x \in S$ can be upgraded to *admissible* [T,P] without *weakening* some $y \in S \cap T$.

Decision problems in AFVs

- 3 natural problems suggested by formal definitions above –
  Top Admissible Scenario (TAS)
  Admissible Upgrade (AU)
  Equilibrated Scenario (ES)
- TAS – is [S,P] top admissible?
- AU – given [S,P] is there *any admissible* upgrade [T,P] for this?
- ES – is [S,P] an equilibrated scenario?

& their complexity

- All of these, in the most general case, turn out to be *computationally hard*: –

  TAS is coNP–complete;

  AU is NP–complete;

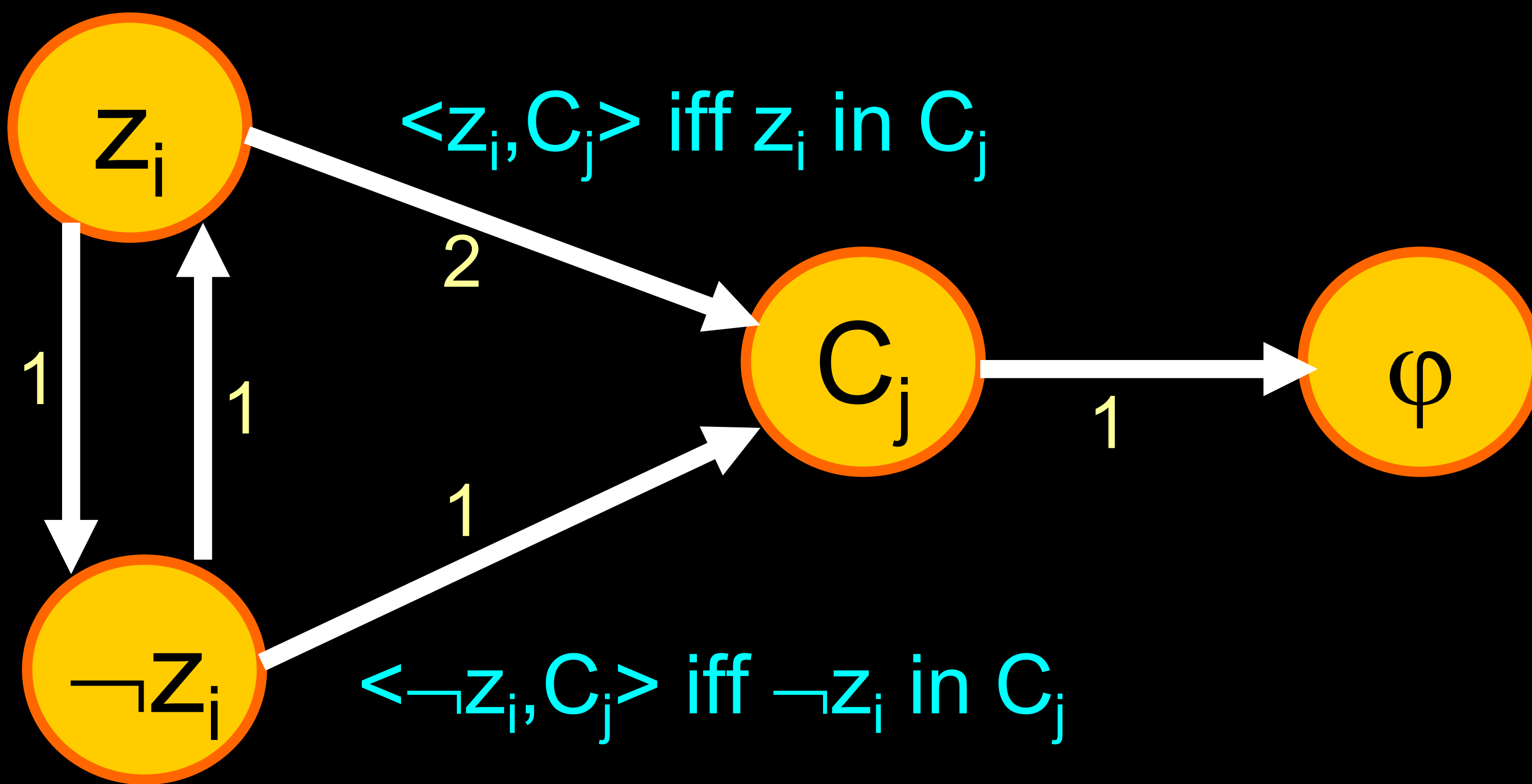
  ES is coNP–complete.
- Proofs use variant of the well-known translation from CNF *formulae* to AFs.
- Need, however, to incorporate elements for *partition* of A and the *relation* R.

TAS is coNP–complete I

- Start from a variant of CNF-SAT: the "*more than one* SAT" problem (MTOS).
- Given a CNF, $\varphi(Z)$, (with *each literal* from Z occuring in *some* clause) & a *satisfying assignment*, α, MTOS reports true if there is a satisfying assignment *in addition to* α.
- MTOS is "easily" shown NP-complete.

$$\varphi(Z) = \{C_1, C_2, \ldots, C_m\} \;\&\; \varphi(\perp, \ldots, \perp)$$

$$\mathrm{MTOS}(\varphi(Z), \langle\perp,\perp,\ldots,\perp\rangle) \Leftrightarrow \neg \mathrm{TAS}([\{\varphi, \neg z_1, \ldots, \neg z_n\}, \{>>, \approx\}])$$



Consequences

- Note that construction holds even when partition of A contains *exactly two* sets.
- That AU is NP-complete is a direct corollary of the reduction establishing TAS to be coNP-complete.
- A slightly more involved construction (but not requiring variant of CNF-SAT) shows that ES is coNP-complete.

Conclusions & Development

- AFVs provide an approach to distinguishing “quality of defence” by specifying an ordering over a partition of the attack relation.
- Via this a number of natural decision questions arise regarding the “optimality” of defensive positions.
- These, however, are computationaly hard.
- One useful development would be to identify cases where efficient algorithms are possible.
- Alternative criteria for capturing “equilibrium” properties are also of some interest.