UNIVERSITÄT LEIPZIG

Expanding Argumentation Frameworks: Enforcing and Monotonicity Results

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Motivation

Dungs AFs are static but argumentation is a dynamic process.

What we have studied?

1. **revising AFs:** How extensions of an AF behave if
   - new arguments and
   - attack relations are added?
   - and/or the underlying semantics are changed?

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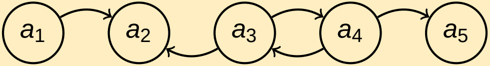
What we have studied?

1. **revising AFs:** How extensions of an AF behave if
   - new arguments and
   - attack relations are added?
   - and/or the underlying semantics are changed?
2. **enforcing problem:** Is it possible (and if so how) to modify a given AF in such a way that a desired set of arguments becomes an extension?

Exemplification

Example 1: revising AFs

Let $\mathcal{A}$ be the following argument graph:



$$\mathcal{E}_{pref}(\mathcal{A}) = \{E_1, E_2\} = \{\{a_1, a_3, a_5\}, \{a_1, a_4\}\}.$$

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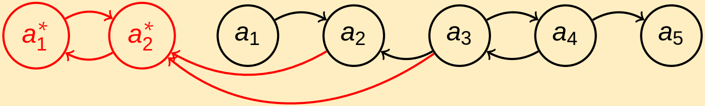

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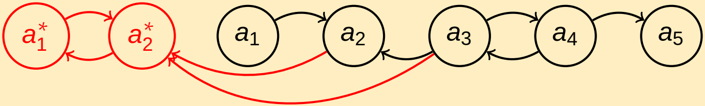

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we observe the following interrelations:

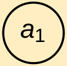
1. the number of extensions increased
2. every old belief set is contained in a new one
3. every new belief set is the union of an old one and a new argument

Exemplification

Example 2: enforcing problem

Consider the following two-agent scenario. The evaluation is given by the grounded semantics.

1^{st} round: Agent A



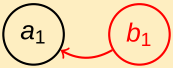
$$\mathcal{E}_{gr}(\mathcal{A}_1) = \{\{a_1\}\}$$

Exemplification

Example 2: enforcing problem

Consider the following two-agent scenario. The evaluation is given by the grounded semantics.

2^{nd} round: Agent B



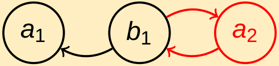
$$\mathcal{E}_{gr}(\mathcal{A}_2) = \{\{b_1\}\}$$

Exemplification

Example 2: enforcing problem

Consider the following two-agent scenario. The evaluation is given by the grounded semantics.

3^{rd} round: Agent A



$$\mathcal{E}_{gr}(\mathcal{A}_3) = \{\emptyset\}$$

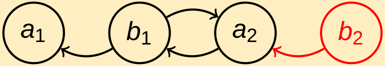
Is it possible for Agent B to get b_1 accepted in the fourth round?

Exemplification

Example 2: enforcing problem

Consider the following two-agent scenario. The evaluation is given by the grounded semantics.

4^{th} round: Agent B



$$\mathcal{E}_{gr}(\mathcal{A}_4) = \{\{b_1, b_2\}\}$$

New Definitions

Normal Expansions

Def.: $\mathcal{A}^*$ is an *expansion* of $\mathcal{A} = (A, R)$ iff $\mathcal{A}^* = (A \cup A^*, R \cup R^*)$ for some nonempty A^* disjoint from A.

An expansion is

1. *normal* ($\mathcal{A} \prec^N \mathcal{A}^*$) iff $\forall ab\ ((a, b) \in R^* \rightarrow a \in A^* \vee b \in A^*)$,

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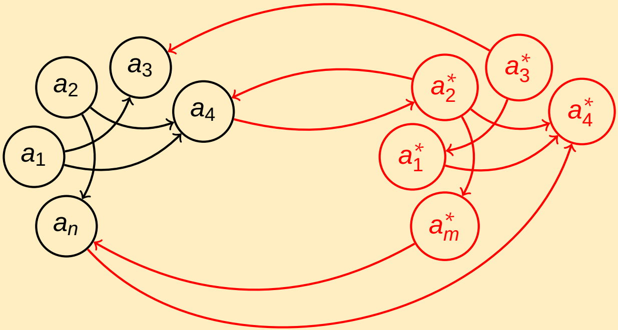

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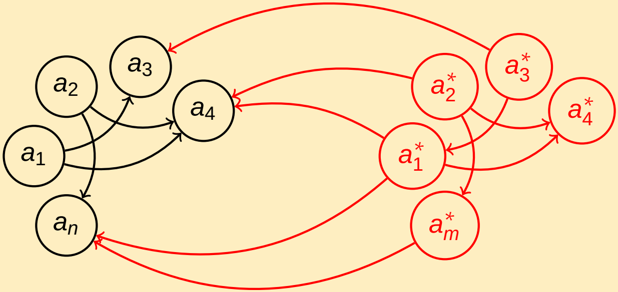

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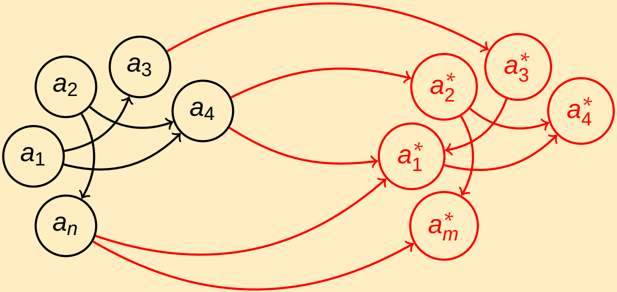

New Definitions

Enforcements

Def.: Given

- an AF $\mathcal{A} = (A, R)$,
- a semantics $\mathcal{S}$ and
- a desired set of arguments E^* w.t.p. $E^* \notin \mathcal{E}_{\mathcal{S}}(\mathcal{A})$.

An *$(\mathcal{A}, \mathcal{S})$-enforcement of E^** is a pair $\mathcal{F} = (\mathcal{A}^*, \mathcal{S}^*)$ such that

- $\mathcal{A}^* = \mathcal{A}$ or $\mathcal{A} \prec^N \mathcal{A}^*$ and
- $E^* \in \mathcal{E}_{\mathcal{S}^*}(\mathcal{A}^*)$ holds.

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$\mathcal{F}$ is called

1. *conservative* if $\mathcal{S} = \mathcal{S}^*$,
2. *conservative strong* if $\mathcal{S} = \mathcal{S}^*$ and $\mathcal{A} \prec^N_S \mathcal{A}^*$,
3. *conservative weak* if $\mathcal{S} = \mathcal{S}^*$ and $\mathcal{A} \prec^N_W \mathcal{A}^*$.

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2. *liberal strong* if $\mathcal{S} \neq \mathcal{S}^*$ and $\mathcal{A} \prec^N_S \mathcal{A}^*$,
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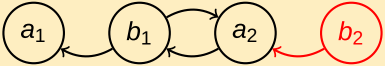
New Definitions

Enforcements

Remember the two-agent scenario from the beginning.

4^{th} round: Agent B



$$\mathcal{E}_{gr}(\mathcal{A}_4) = \{\{b_1, b_2\}\}$$

This is a conservative strong enforcement of $\{b_1, b_2\}$.

Possibility Results

Theorem 1 (conservative strong enforcement)

Let

- $\mathcal{A} = (A, R)$ be an AF
- $\mathcal{S}$ a semantics and
- $C \subseteq A$ a conflict-free set w.t.p. $C \notin \mathcal{E}_\sigma(\mathcal{A})$.

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- $C \subseteq A$ a conflict-free set w.t.p. $C \notin \mathcal{E}_\sigma(\mathcal{A})$.

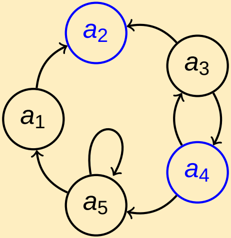
There is a conservative strong enforcement $\mathcal{F} = (\mathcal{A}^*, \mathcal{S})$ of C^* w.t.p.:

- $|C^* \backslash C| = 1$ and
- C^* is the unique extension of $\mathcal{A}^*$ (for $\mathcal{S} \in \{st, pr, co, gr, id\}$) or
- set-inclusion maximal extension for admissible semantics.

Possibility Results

Example 3 - Standard Construction

Let $\mathcal{A}$ be the following argument graph:



$C = \{a_2, a_4\} \notin \mathcal{E}_\sigma(\mathcal{A})$ for all $\sigma \in \{st, ad, pr, co, gr, id\}$

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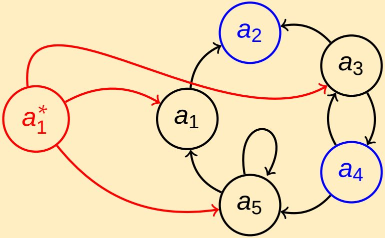

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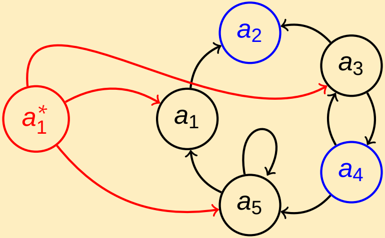

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open question: Minimal change?

Impossibility Results

Recap - Abstract Principles [Baroni/Giacomin]

Def.: A semantics $\mathcal{S}$ satisfies

1. *admissibility*,
2. *reinstatement*,
3. *conflict-freeness*

if and only if for any argumentation framework $\mathcal{A} \in \mathcal{D}_{\mathcal{S}}$ and any extension $E \in \mathcal{E}_{\mathcal{S}}(\mathcal{A})$ it holds that:

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1. $\forall a \ (a \in E \rightarrow \forall b \ (b \in A \land (b,a) \in R \rightarrow (E,\{b\}) \bar{\in} R))$,
   "E defends all its elements."
2. $\forall a \ (\forall b \ (b \in A \land (b,a) \in R \rightarrow (E,\{b\}) \bar{\in} R) \rightarrow a \in E)$,
   "Every argument defended by E is an element of E."
3. $(E,E) \bar{\notin} R$.

Impossibility Results

Theorem 2 (exchanging believed with unattacking arguments)

Given an AF $\mathcal{A} = (A, R)$ and

- a semantics $\mathcal{S}$ satisfying reinstatement,
- a semantics $\mathcal{S}^*$, satisfying admissibility and conflict-freeness and
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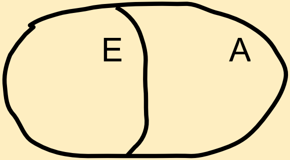
There is no enforcement $\mathcal{F} = (\mathcal{A}^*, \mathcal{S}^*)$ of E^* if

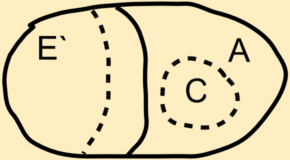
- $E^* = E' \cup C$, given that
- $E' \subseteq E$, (subset of the *old* extension)
- $\emptyset \neq C \subseteq A \backslash E$ and (formely unaccepted arguments)
- $(C, A \backslash \{E' \cup C\}) \notin R$. (no attacks to *outer* arguments)

Impossibility Results

Theorem 2 (illustration)



$E \in \mathcal{E}_{\mathcal{S}}(\mathcal{A})$



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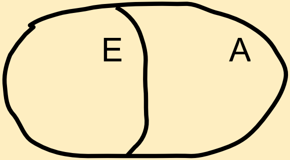

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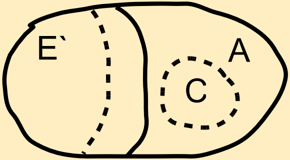

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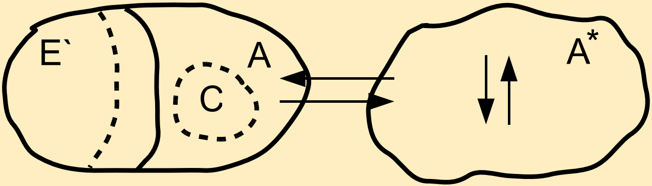

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Given an AF $\mathcal{A} = (A, R)$ and

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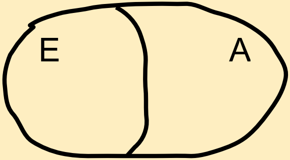

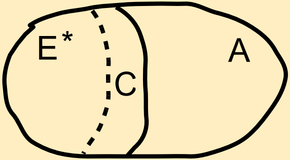

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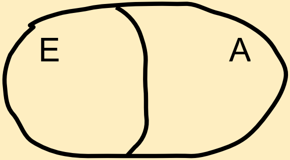

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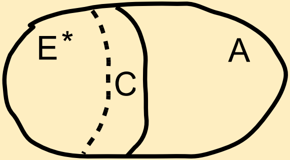

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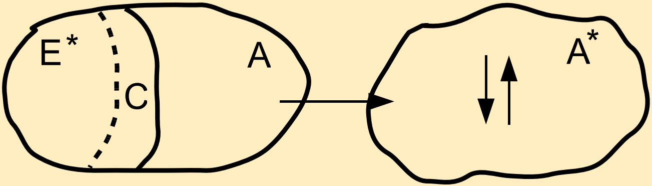

$E^* = E \backslash C \notin \mathcal{E}_{\mathcal{S}^*}(\mathcal{A}^*)$

Monotonicity

Recap - Abstract Principles [Baroni/Giacomin]

Def.: A semantics $\mathcal{S}$ satisfies the *directionality* principle if and only if for any argumentation framework $\mathcal{A} \in \mathcal{D}_{\mathcal{S}}$ and any unattacked set $U \in \mathcal{US}(\mathcal{A})$ it holds that:

- $\mathcal{E}_{\mathcal{S}}(\mathcal{A}_{\downarrow U}) = \{(E \cap U) \mid E \in \mathcal{E}_{\mathcal{S}}(\mathcal{A})\}$.

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Theorem 4 (Monotonicity)

Given an AF $\mathcal{A} = (A, R)$ and a semantics $\mathcal{S}$ satisfying directionality, then for all weak expansions $\mathcal{A}^*$ of $\mathcal{A}$ the following holds:

1. $|\mathcal{E}_\mathcal{S}(\mathcal{A})| \leq |\mathcal{E}_\mathcal{S}(\mathcal{A}^*)|$,
2. $\forall E \in \mathcal{E}_\mathcal{S}(\mathcal{A})\ \exists E^* \in \mathcal{E}_\mathcal{S}(\mathcal{A}^*) : E \subseteq E^*$ and
3. $\forall E^* \in \mathcal{E}_\mathcal{S}(\mathcal{A}^*)\ \exists E_i \in \mathcal{E}_\mathcal{S}(\mathcal{A})\ \exists A_i^* \subseteq A^* : E^* = E_i \cup A_i^*$.

Monotonicity

Corrollary 1

Given the same assumptions as in Theorem 4, then

1. $\bigcup_{E \in \mathcal{E}_{\mathcal{S}}(\mathcal{A})} E \subseteq \bigcup_{E^* \in \mathcal{E}_{\mathcal{S}}(\mathcal{A}^*)} E^*$ (credulously justified args persist),
2. $\bigcap_{E \in \mathcal{E}_{\mathcal{S}}(\mathcal{A})} E \subseteq \bigcap_{E^* \in \mathcal{E}_{\mathcal{S}}(\mathcal{A}^*)} E^*$ (skeptically justified args persist).

Monotonicity

Expansion Chain

Def.: Let $C = \langle \mathcal{A}_0, \ldots, \mathcal{A}_n \rangle$ be a sequence of AFs, $\mathcal{A}$ an AF. C is called expansion chain of $\mathcal{A}$ iff

1. $\mathcal{A} = \mathcal{A}_n$ and
2. $\mathcal{A}_i \prec^N \mathcal{A}_{i+1}$ ($\mathcal{A}_{i+1}$ is a normal expansion of $\mathcal{A}_i$) for all i: $0 \leq i \leq n-1$.

C is called weak (resp. strong) if all expansions in the chain are weak (resp. strong).

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C is called weak (resp. strong) if all expansions in the chain are weak (resp. strong).

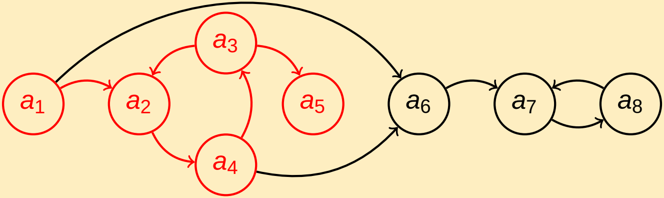
Corrollary 2

Let $C = \langle \mathcal{A}_0, \ldots, \mathcal{A}_n \rangle$ be a weak expansion chain of $\mathcal{A}$, and let i be the smallest integer such that $\mathcal{A}_i$ covers a.Given that $\mathcal{S}$ satisfies the directionality principle, we get: a is in some/all extensions of $\mathcal{A}$ iff a is in some/all extensions of $\mathcal{A}_i$.

Monotonicity

Example - Checking Acceptability

Acceptability of a_5 in the red colored AF decides its acceptability in the whole AF.



Open Questions

1. Minimal changes (enforcing problem)?
2. Monotonicity results for strong expansions?
3. Algorithm for detecting weak expansion chains (in process)?

Open Questions

1. Minimal changes (enforcing problem)?
2. Monotonicity results for strong expansions?
3. Algorithm for detecting weak expansion chains (in process)?

Thank you for your attention!

Questions?