Generalizing stable semantics by preferences

Leila Amgoud Srdjan Vesic

IRIT - Toulouse

amgoud@irit.fr vesic@irit.fr

Abstract argumentation frameworks (Dung '95)

- An argumentation framework is a pair $(\mathcal{A}, \mathcal{R})$ where:
  - $\mathcal{A}$ is a set of arguments
  - $\mathcal{R}$ is an attack relation ($\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$)

- Which arguments to accept together? $\Rightarrow$ semantics
  - Stable: conflict-free set attacking any argument outside the set
  - Preferred: maximal conflict-free and self-defending set
  - Grounded: minimal conflict-free set containing all arguments it defends

Example

Let $\mathcal{F}_1$ be as depicted below:

- $\{a\}, \{b\}$ are the two stable, preferred extensions of $\mathcal{F}_1$
- $\{\}$ is the grounded extension of $\mathcal{F}_1$

Need for preferences

- An argument may be stronger than another:
  - is built from more certain information
  - refers to important goals
  - promotes more important value
  - ...

Need for preferences

- An argument may be stronger than another:
  - is built from more certain information
  - refers to important goals
  - promotes more important value
  - ...
- Need to take into account the strengths of arguments (captured by a preference relation $\geq \subseteq \mathcal{A} \times \mathcal{A}$)

Preference-based argumentation frameworks

(Amgoud & Cayrol'02, Bench-Capon'03, Sanjay'09)

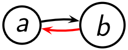
- A preference-based framework (PAF) is a tuple $(\mathcal{A}, \mathcal{R}, \geq)$

Preference-based argumentation frameworks

(Amgoud & Cayrol'02, Bench-Capon'03, Sanjay'09)

- A preference-based framework (PAF) is a tuple $(\mathcal{A}, \mathcal{R}, \geq)$

$\mathcal{F}_1$



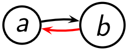
$a > b$

Preference-based argumentation frameworks

(Amgoud & Cayrol'02, Bench-Capon'03, Sanjay'09)

- A preference-based framework (PAF) is a tuple $(\mathcal{A}, \mathcal{R}, \geq)$

$\mathcal{F}_1$



$\Longrightarrow$

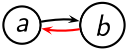
$a > b$

Preference-based argumentation frameworks

(Amgoud & Cayrol'02, Bench-Capon'03, Sanjay'09)

- A preference-based framework (PAF) is a tuple $(\mathcal{A}, \mathcal{R}, \geq)$

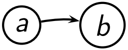
$\mathcal{F}_1$



$a > b$

$\Longrightarrow$

$\mathcal{F}_2$

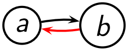


Preference-based argumentation frameworks

(Amgoud & Cayrol'02, Bench-Capon'03, Sanjay'09)

- A preference-based framework (PAF) is a tuple $(\mathcal{A}, \mathcal{R}, \geq)$

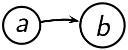
$\mathcal{F}_1$



$a > b$

$\Longrightarrow$

$\mathcal{F}_2$



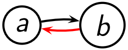
$\Downarrow$

Preference-based argumentation frameworks

(Amgoud & Cayrol'02, Bench-Capon'03, Sanjay'09)

- A preference-based framework (PAF) is a tuple $(\mathcal{A}, \mathcal{R}, \geq)$

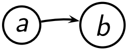
$\mathcal{F}_1$



$a > b$

$\Longrightarrow$

$\mathcal{F}_2$



$\Downarrow$

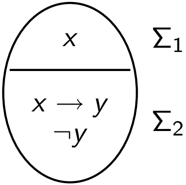
$\{a\}$ is a stable extension of $\mathcal{F}_2$

. . . but things can be wrong when $\mathcal{R}$ is not symmetric

- An argument is a minimal proof
- $a\mathcal{R}b$ iff the conclusion of a contradicts a formula in the support of b

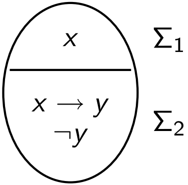
. . . but things can be wrong when $\mathcal{R}$ is not symmetric

- An argument is a minimal proof
- $a\mathcal{R}b$ iff the conclusion of a contradicts a formula in the support of b



. . . but things can be wrong when $\mathcal{R}$ is not symmetric

- An argument is a minimal proof
- $a\mathcal{R}b$ iff the conclusion of a contradicts a formula in the support of b



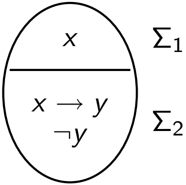
$a_1 : (\{x\}, x)$ $\quad$ $a_2 : (\{\neg y\}, \neg y)$

$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $\quad$ $a_4 : (\{x, \neg y\}, x \wedge \neg y)$

$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $\quad$ $a_6 : (\{x, x \rightarrow y\}, y)$

... but things can be wrong when $\mathcal{R}$ is not symmetric

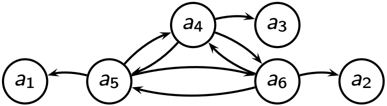
- An argument is a minimal proof
- $a\mathcal{R}b$ iff the conclusion of a contradicts a formula in the support of b



$a_1 : (\{x\}, x)$ $\quad a_2 : (\{\neg y\}, \neg y)$

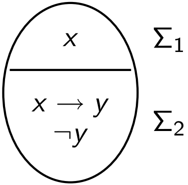
$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $\quad a_4 : (\{x, \neg y\}, x \wedge \neg y)$

$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $\quad a_6 : (\{x, x \rightarrow y\}, y)$



…but things can be wrong when $\mathcal{R}$ is not symmetric

- An argument is a minimal proof
- $a\mathcal{R}b$ iff the conclusion of a contradicts a formula in the support of b

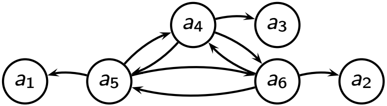


$a >_{wlp} b$ iff the weakest formula in the support of a is stronger than the weakest formula in the support of b

$a_1 : (\{x\}, x)$ $\quad$ $a_2 : (\{\neg y\}, \neg y)$

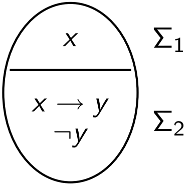
$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $\quad$ $a_4 : (\{x, \neg y\}, x \wedge \neg y)$

$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $\quad$ $a_6 : (\{x, x \rightarrow y\}, y)$



. . . but things can be wrong when $\mathcal{R}$ is not symmetric

- An argument is a minimal proof
- $a\mathcal{R}b$ iff the conclusion of a contradicts a formula in the support of b

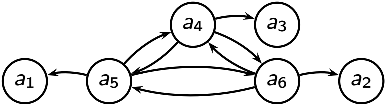


$a >_{wlp} b$ iff the weakest formula in the support of a is stronger than the weakest formula in the support of b

$a_1 : (\{x\}, x)$ $\quad$ $a_2 : (\{\neg y\}, \neg y)$

$a_3 : (\{x \to y\}, x \to y)$ $\quad$ $a_4 : (\{x, \neg y\}, x \wedge \neg y)$

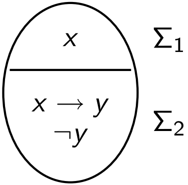
$a_5 : (\{\neg y, x \to y\}, \neg x)$ $\quad$ $a_6 : (\{x, x \to y\}, y)$



$a_1 > a_2, a_3, a_4, a_5, a_6$

. . . but things can be wrong when $\mathcal{R}$ is not symmetric

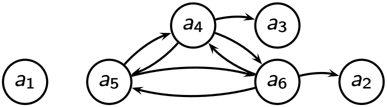
- An argument is a minimal proof
- $a\mathcal{R}b$ iff the conclusion of a contradicts a formula in the support of b



$a_1 : (\{x\}, x)$ $\quad a_2 : (\{\neg y\}, \neg y)$

$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $\quad a_4 : (\{x, \neg y\}, x \wedge \neg y)$

$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $\quad a_6 : (\{x, x \rightarrow y\}, y)$

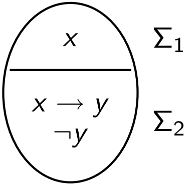


$a >_{wlp} b$ iff the weakest formula in the support of a is stronger than the weakest formula in the support of b

$a_1 > a_2, a_3, a_4, a_5, a_6$

. . . but things can be wrong when $\mathcal{R}$ is not symmetric

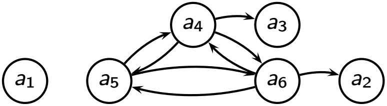
- Three stable extensions:
- $\mathcal{E}_1 = \{a_1, a_2, a_3, a_5\}$, $\mathcal{E}_2 = \{a_1, a_2, a_4\}$, $\mathcal{E}_3 = \{a_1, a_3, a_6\}$



$a_1 : (\{x\}, x)$ $\quad a_2 : (\{\neg y\}, \neg y)$

$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $\quad a_4 : (\{x, \neg y\}, x \wedge \neg y)$

$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $\quad a_6 : (\{x, x \rightarrow y\}, y)$

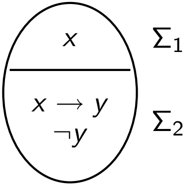


$a >_{wlp} b$ iff the weakest formula in the support of a is stronger than the weakest formula in the support of b

$a_1 > a_2, a_3, a_4, a_5, a_6$

. . . but things can be wrong when $\mathcal{R}$ is not symmetric

- $\mathcal{E}_1$: conflicting arguments, contradictory conclusions, inconsistent base
- $\mathcal{E}_1 = \{a_1, a_2, a_3, a_5\}$, $\mathcal{E}_2 = \{a_1, a_2, a_4\}$, $\mathcal{E}_3 = \{a_1, a_3, a_6\}$

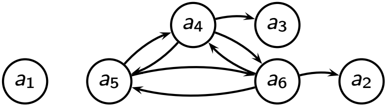


$a >_{wlp} b$ iff the weakest formula in the support of a is stronger than the weakest formula in the support of b

$a_1 : (\{x\}, x)$ $\quad a_2 : (\{\neg y\}, \neg y)$

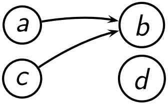
$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $\quad a_4 : (\{x, \neg y\}, x \wedge \neg y)$

$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $\quad a_6 : (\{x, x \rightarrow y\}, y)$



$a_1 > a_2, a_3, a_4, a_5, a_6$

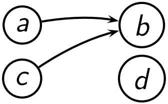
Need for comparing sets of arguments



$a \approx c \quad a > b$

$c > b \quad d > b$

Need for comparing sets of arguments

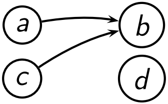


$a \approx c \quad a > b$

$c > b \quad d > b$

- Stable / preferred / grounded extension: $\{a, c, d\}$

Need for comparing sets of arguments



$a \approx c \quad a > b$

$c > b \quad d > b$

- Stable / preferred / grounded extension: $\{a, c, d\}$
- It is impossible to conclude that:
  - $\{a, c\} \succ \{b\}$
  - $\{d\} \succ \{b\}$
  - ...

A new approach for preference-based argumentation

Idea: to define new acceptability semantics that:

- are based on preferences and attacks between arguments

A new approach for preference-based argumentation

Idea: to define new acceptability semantics that:

- are based on preferences and attacks between arguments
- generalize Dung's semantics

A new approach for preference-based argumentation

Idea: to define new acceptability semantics that:

- are based on preferences and attacks between arguments
- generalize Dung's semantics
- ensure conflict-free extensions

A new approach for preference-based argumentation

Idea: to define new acceptability semantics that:

- are based on preferences and attacks between arguments
- generalize Dung's semantics
- ensure conflict-free extensions
- allow to compare any pair of subsets of arguments

A new approach for preference-based argumentation

Definition (New semantics)

Let $(\mathcal{A}, \mathcal{R}, \geq)$ be a PAF. A semantics is defined by a dominance relation $\succeq \subseteq \mathcal{P}(\mathcal{A}) \times \mathcal{P}(\mathcal{A})$.

The extensions of $(\mathcal{A}, \mathcal{R}, \geq)$ are the maximal elements of $\succeq$.

Definition (Maximal element)

$\mathcal{E} \in \mathcal{P}(\mathcal{A})$ is a maximal element of a dominance relation $\succeq$ iff $\forall \mathcal{E}' \in \mathcal{P}(\mathcal{A})$, $\mathcal{E} \succeq \mathcal{E}'$.

$\succeq_{max}$ = the set of all maximal elements wrt $\succeq$.

When does a dominance relation generalize a semantics?

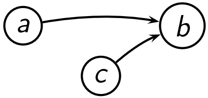
Definition

A dominance relation $\succeq \subseteq \mathcal{P}(A) \times \mathcal{P}(A)$ generalizes semantics x iff for all PAF $(\mathcal{A}, \mathcal{R}, \geq)$ if $\nexists a, b \in \mathcal{A}$ s.t. $a \mathcal{R} b$ and $b > a$, then $\succeq_{max}$ is exactly the set of extensions of $(\mathcal{A}, \mathcal{R})$ wrt semantics x.

When does a dominance relation generalize a semantics?

Definition

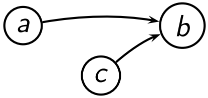
A dominance relation $\succeq \subseteq \mathcal{P}(A) \times \mathcal{P}(A)$ generalizes semantics x iff for all PAF $(\mathcal{A}, \mathcal{R}, \geq)$ if $\nexists a, b \in \mathcal{A}$ s.t. $a\mathcal{R}b$ and $b > a$, then $\succeq_{max}$ is exactly the set of extensions of $(\mathcal{A}, \mathcal{R})$ wrt semantics x.



When does a dominance relation generalize a semantics?

Definition

A dominance relation $\succeq \subseteq \mathcal{P}(A) \times \mathcal{P}(A)$ generalizes semantics x iff for all PAF $(\mathcal{A}, \mathcal{R}, \geq)$ if $\nexists a, b \in \mathcal{A}$ s.t. $a\mathcal{R}b$ and $b > a$, then $\succeq_{max}$ is exactly the set of extensions of $(\mathcal{A}, \mathcal{R})$ wrt semantics x.



$a > b$, $a > c$

Generalizing stable semantics with preferences

(Amgoud & Vesic, IJCAI'09)

Definition (Pref-stable semantics)

Let $\mathcal{T} = (\mathcal{A}, \mathcal{R}, \geq)$ be a PAF and $\mathcal{E}, \mathcal{E}' \in \mathcal{P}(\mathcal{A})$. $\mathcal{E} \succeq_{st} \mathcal{E}'$ iff:

- $\mathcal{E}$ is conflict-free and $\mathcal{E}'$ is not conflict-free, or
- $\mathcal{E}$ and $\mathcal{E}'$ are conflict-free and $\forall a' \in \mathcal{E}' \setminus \mathcal{E}$, $\exists a \in \mathcal{E} \setminus \mathcal{E}'$ s.t. $(a\mathcal{R}a'$ and $a' \not> a)$ or $(a > a')$

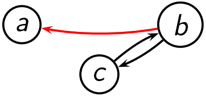
Generalizing stable semantics with preferences

(Amgoud & Vesic, IJCAI'09)

Definition (Pref-stable semantics)

Let $\mathcal{T} = (\mathcal{A}, \mathcal{R}, \geq)$ be a PAF and $\mathcal{E}, \mathcal{E}' \in \mathcal{P}(\mathcal{A})$. $\mathcal{E} \succeq_{st} \mathcal{E}'$ iff:

- $\mathcal{E}$ is conflict-free and $\mathcal{E}'$ is not conflict-free, or
- $\mathcal{E}$ and $\mathcal{E}'$ are conflict-free and $\forall a' \in \mathcal{E}' \setminus \mathcal{E}$, $\exists a \in \mathcal{E} \setminus \mathcal{E}'$ s.t. $(a \mathcal{R} a'$ and $a' \not> a)$ or $(a > a')$



$a > b$

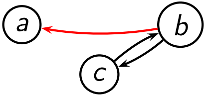
Generalizing stable semantics with preferences

(Amgoud & Vesic, IJCAI'09)

Definition (Pref-stable semantics)

Let $\mathcal{T} = (\mathcal{A}, \mathcal{R}, \geq)$ be a PAF and $\mathcal{E}, \mathcal{E}' \in \mathcal{P}(\mathcal{A})$. $\mathcal{E} \succeq_{st} \mathcal{E}'$ iff:

- $\mathcal{E}$ is conflict-free and $\mathcal{E}'$ is not conflict-free, or
- $\mathcal{E}$ and $\mathcal{E}'$ are conflict-free and $\forall a' \in \mathcal{E}' \setminus \mathcal{E}$, $\exists a \in \mathcal{E} \setminus \mathcal{E}'$ s.t. $(a\mathcal{R}a'$ and $a' \not> a)$ or $(a > a')$



$a > b$

- $\{a\} \succ_{st} \{b\}$
- $\emptyset \succ_{st} \{a, b, c\}$
- $\{b\} \succ_{st} \emptyset$
- ...
- $\succeq_{st,max} = \{\{a, c\}\}$

Generalizing stable semantics with preferences

(Amgoud & Vesic, IJCAI'09)

Definition (Pref-stable semantics)

Let $\mathcal{T} = (\mathcal{A}, \mathcal{R}, \geq)$ be a PAF and $\mathcal{E}, \mathcal{E}' \in \mathcal{P}(\mathcal{A})$. $\mathcal{E} \succeq_{st} \mathcal{E}'$ iff:

- $\mathcal{E}$ is conflict-free and $\mathcal{E}'$ is not conflict-free, or
- $\mathcal{E}$ and $\mathcal{E}'$ are conflict-free and $\forall a' \in \mathcal{E}' \setminus \mathcal{E}$, $\exists a \in \mathcal{E} \setminus \mathcal{E}'$ s.t. $(a\mathcal{R}a'$ and $a' \not> a)$ or $(a > a')$

Theorem (1)

Let $\mathcal{T} = (\mathcal{A}, \mathcal{R}, \geq)$ be a PAF.

- *The relation $\succeq_{st}$ generalizes stable semantics.*
- *For all $\mathcal{E} \in \succeq_{st,max}$, $\mathcal{E}$ is a maximal conflict-free subset of $\mathcal{A}$.*

How to choose a dominance relation?

- Are there other relations that generalize this semantics?

How to choose a dominance relation?

- Are there other relations that generalize this semantics?
- If yes,

How to choose a dominance relation?

- Are there other relations that generalize this semantics?
- If yes,
  - what are the differences between them?

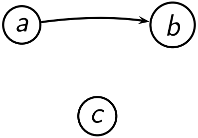
How to choose a dominance relation?

- Are there other relations that generalize this semantics?
- If yes,
  - what are the differences between them?
  - how to compare them?

How to choose a dominance relation?

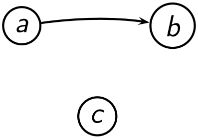
- Are there other relations that generalize this semantics?
- If yes,
  - what are the differences between them?
  - how to compare them?
  - are they all meaningful?

How to choose a dominance relation?

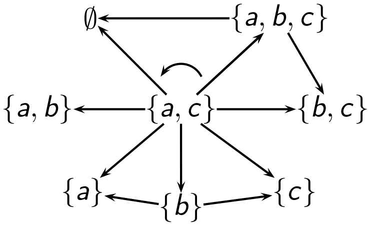


$c > b$

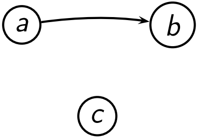
How to choose a dominance relation?



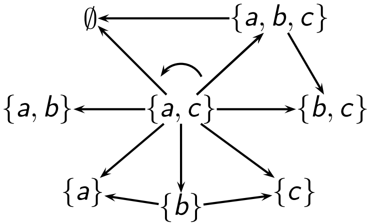
$c > b$



How to choose a dominance relation?

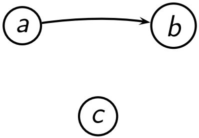


$c > b$

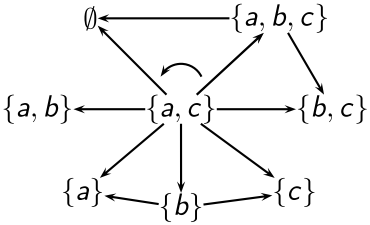


- This relation generalizes stable semantics

How to choose a dominance relation?

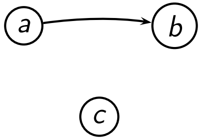


$c > b$

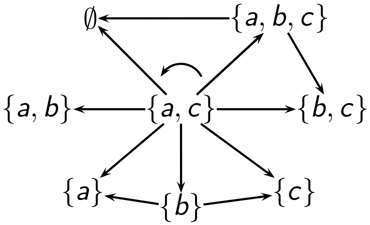


- This relation generalizes stable semantics
- However:
  - $\{a, b, c\} \succ \{b, c\}$

How to choose a dominance relation?

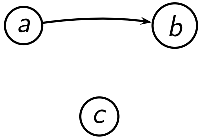


$c > b$

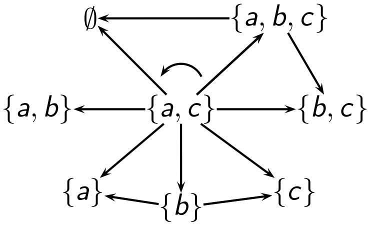


- This relation generalizes stable semantics
- However:
    - $\{a, b, c\} \succ \{b, c\}$
    - $\{b\} \succ \{a\}$

How to choose a dominance relation?



$c > b$



- This relation generalizes stable semantics
- However:
  - $\{a, b, c\} \succ \{b, c\}$
  - $\{b\} \succ \{a\}$
  - $\{b\} \succ \{c\}$

Postulates

Postulate (1)

$$\frac{\mathcal{E} \in \mathcal{CF} \qquad \mathcal{E}' \notin \mathcal{CF}}{\mathcal{E} \succ \mathcal{E}'}$$

Postulates

Postulate (1)

$$\frac{\mathcal{E} \in \mathcal{CF} \qquad \mathcal{E}' \notin \mathcal{CF}}{\mathcal{E} \succ \mathcal{E}'}$$

Postulate (2)

Let $\mathcal{E}, \mathcal{E}' \in \mathcal{CF}$.

$$\frac{\mathcal{E} \succeq \mathcal{E}'}{\mathcal{E} \setminus \mathcal{E}' \succeq \mathcal{E}' \setminus \mathcal{E}}$$

$$\frac{\mathcal{E} \setminus \mathcal{E}' \succeq \mathcal{E}' \setminus \mathcal{E}}{\mathcal{E} \succeq \mathcal{E}'}$$

Postulates

Postulate (1)

$$\frac{\mathcal{E} \in \mathcal{CF} \qquad \mathcal{E}' \notin \mathcal{CF}}{\mathcal{E} \succ \mathcal{E}'}$$

Postulate (2)

Let $\mathcal{E}, \mathcal{E}' \in \mathcal{CF}$.

$$\frac{\mathcal{E} \succeq \mathcal{E}'}{\mathcal{E} \setminus \mathcal{E}' \succeq \mathcal{E}' \setminus \mathcal{E}}$$

$$\frac{\mathcal{E} \setminus \mathcal{E}' \succeq \mathcal{E}' \setminus \mathcal{E}}{\mathcal{E} \succeq \mathcal{E}'}$$

Postulate (3)

Let $\mathcal{E}, \mathcal{E}' \in \mathcal{CF}$ *and* $\mathcal{E} \cap \mathcal{E}' = \emptyset$.

$$\frac{(\exists a' \in \mathcal{E}')(\forall a \in \mathcal{E}) \quad \neg(a\mathcal{R}a' \wedge a' \not\geq a) \wedge a \not\geq a'}{\neg(\mathcal{E} \succeq \mathcal{E}')}$$

Postulates

Postulate (1)

$$\frac{\mathcal{E} \in \mathcal{CF} \qquad \mathcal{E}' \notin \mathcal{CF}}{\mathcal{E} \succ \mathcal{E}'}$$

Postulate (2)

Let $\mathcal{E}, \mathcal{E}' \in \mathcal{CF}$.

$$\frac{\mathcal{E} \succeq \mathcal{E}'}{\mathcal{E} \setminus \mathcal{E}' \succeq \mathcal{E}' \setminus \mathcal{E}}$$

$$\frac{\mathcal{E} \setminus \mathcal{E}' \succeq \mathcal{E}' \setminus \mathcal{E}}{\mathcal{E} \succeq \mathcal{E}'}$$

Postulate (3)

Let $\mathcal{E}, \mathcal{E}' \in \mathcal{CF}$ *and* $\mathcal{E} \cap \mathcal{E}' = \emptyset$.

$$\frac{(\exists a' \in \mathcal{E}')(\forall a \in \mathcal{E}) \; \neg(a\mathcal{R}a' \wedge a' \not> a) \wedge a \not> a'}{\neg(\mathcal{E} \succeq \mathcal{E}')}$$

Postulate (4)

Let $\mathcal{E}, \mathcal{E}' \in \mathcal{CF}$ *and* $\mathcal{E} \cap \mathcal{E}' = \emptyset$.

$$\frac{(\forall a' \in \mathcal{E}')(\exists a \in \mathcal{E}) \; (a\mathcal{R}a' \wedge a' \not> a) \text{ or } (a\mathcal{R}a' \wedge a > a')}{\mathcal{E} \succeq \mathcal{E}'}$$

Properties of relations satisfying the postulates

Theorem (2)

If $\succeq$ satisfies Postulates 1-4, then $\succeq$ generalizes stable semantics.

Properties of relations satisfying the postulates

Theorem (2)

If $\succeq$ satisfies Postulates 1-4, then $\succeq$ generalizes stable semantics.

Theorem (3)

If $\succeq$ satisfies Postulates 1-4, then any element in $\succeq_{max}$ is conflict-free.

Properties of relations satisfying the postulates

Theorem (2)

If $\succeq$ satisfies Postulates 1-4, then $\succeq$ generalizes stable semantics.

Theorem (3)

If $\succeq$ satisfies Postulates 1-4, then any element in $\succeq_{max}$ is conflict-free.

Theorem (4)

If $\succeq$ and $\succeq'$ both satisfy Postulates 1-4, then $\succeq_{max} = \succeq'_{max}$.

The upper and lower bounds

Definition (The most general pref-stable relation)

Let $\mathcal{T} = (\mathcal{A}, \mathcal{R}, \geq)$ be a PAF and $\mathcal{E}, \mathcal{E}' \in \mathcal{P}(\mathcal{A})$. $\mathcal{E} \succeq_g \mathcal{E}'$ iff:

- $\mathcal{E} \in \mathcal{CF}$ and $\mathcal{E}' \notin \mathcal{CF}$, or
- $\mathcal{E}, \mathcal{E}' \in \mathcal{CF}$ and $\forall a' \in \mathcal{E}' \setminus \mathcal{E}$, $\exists a \in \mathcal{E} \setminus \mathcal{E}'$ s.t. ($a \mathcal{R} a'$ and $a' \not> a$) or ($a' \mathcal{R} a \wedge a > a'$).

The upper and lower bounds

Definition (The most general pref-stable relation)

Let $\mathcal{T} = (\mathcal{A}, \mathcal{R}, \geq)$ be a PAF and $\mathcal{E}, \mathcal{E}' \in \mathcal{P}(\mathcal{A})$. $\mathcal{E} \succeq_g \mathcal{E}'$ iff:

- $\mathcal{E} \in \mathcal{CF}$ and $\mathcal{E}' \notin \mathcal{CF}$, or
- $\mathcal{E}, \mathcal{E}' \in \mathcal{CF}$ and $\forall a' \in \mathcal{E}' \setminus \mathcal{E}$, $\exists a \in \mathcal{E} \setminus \mathcal{E}'$ s.t. $(a\mathcal{R}a'$ and $a' \not> a)$ or $(a'\mathcal{R}a \wedge a > a')$.

Definition (The most specific pref-stable relation)

Let $\mathcal{T} = (\mathcal{A}, \mathcal{R}, \geq)$ be a PAF and $\mathcal{E}, \mathcal{E}' \in \mathcal{P}(\mathcal{A})$. $\mathcal{E} \succeq_s \mathcal{E}'$ iff:

- $\mathcal{E}' \notin \mathcal{CF}$, or
- $\mathcal{E}, \mathcal{E}' \in \mathcal{CF}$ and $\forall a' \in \mathcal{E}' \setminus \mathcal{E}$, $\exists a \in \mathcal{E} \setminus \mathcal{E}'$ s.t. $(a\mathcal{R}a'$ and $a' \not> a)$ or $(a > a')$.

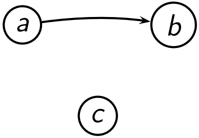
The upper and lower bounds

Theorem (5)

Let $\mathcal{T} = (\mathcal{A}, \mathcal{R}, \geq)$ be a PAF and $\mathcal{E}, \mathcal{E}' \in \mathcal{P}(\mathcal{A})$.
Let $\succeq \subseteq \mathcal{P}(\mathcal{A}) \times \mathcal{P}(\mathcal{A})$ satisfy Postulates 1-4.

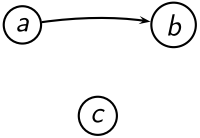
- *If $\mathcal{E} \succeq_g \mathcal{E}'$ then $\mathcal{E} \succeq \mathcal{E}'$.*
- *If $\mathcal{E} \succeq \mathcal{E}'$ then $\mathcal{E} \succeq_s \mathcal{E}'$.*

Differences between pref-stable relations



$a > c$

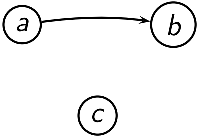
Differences between pref-stable relations



$a > c$

- $\{a\} \succ_{st} \{c\}$
- $\{a\} \succ_{s} \{c\}$
- $\neg(\{a\} \succeq_{g} \{c\})$, $\neg(\{c\} \succeq_{g} \{a\})$

Differences between pref-stable relations



$a > c$

- $\{a\} \succ_{st} \{c\}$
- $\{a\} \succ_{s} \{c\}$
- $\neg(\{a\} \succeq_g \{c\})$, $\neg(\{c\} \succeq_g \{a\})$

- $\{a, b, c\} \succeq_s \{a, b\}$, $\{a, b\} \succeq_s \{a, b, c\}$
- $\neg(\{a, b, c\} \succeq_{st} \{a, b\})$, $\neg(\{a, b\} \succeq_{st} \{a, b, c\})$
- $\neg(\{a, b, c\} \succeq_g \{a, b\})$, $\neg(\{a, b\} \succeq_g \{a, b, c\})$

Characterizing pref-stable semantics

- How to compute pref-stable extensions without comparing all pairs of subsets of $\mathcal{A}$?

Characterizing pref-stable semantics

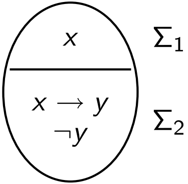
- How to compute pref-stable extensions without comparing all pairs of subsets of $\mathcal{A}$?

Theorem (6)

Let $\succeq$ verify Postulates 1-4.
Let $\mathcal{R}' = \{(a, b) \in \mathcal{A} \times \mathcal{A} \mid (a\mathcal{R}b \wedge b \not> a) \vee (b\mathcal{R}a \wedge a > b)\}$.
Elements of $\succeq_{max}$ are exactly the stable extensions of $(\mathcal{A}, \mathcal{R}')$.

Calculating pref-stable extensions: Example

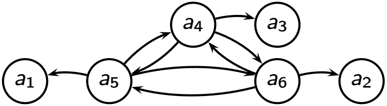
- An argument is a minimal proof
- $a\mathcal{R}b$ iff the conclusion of a contradicts a formula in the support of b
- $\geq$ = the weakest link principle



$a_1 : (\{x\}, x)$ $\quad a_2 : (\{\neg y\}, \neg y)$

$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $\quad a_4 : (\{x, \neg y\}, x \wedge \neg y)$

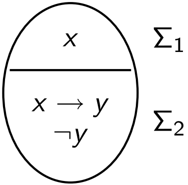
$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $\quad a_6 : (\{x, x \rightarrow y\}, y)$



$a_1 > a_2, a_3, a_4, a_5, a_6$

Calculating pref-stable extensions: Example

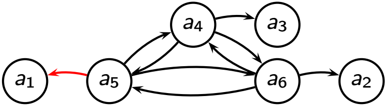
- An argument is a minimal proof
- $a\mathcal{R}b$ iff the conclusion of a contradicts a formula in the support of b
- $\geq$ = the weakest link principle



$a_1 : (\{x\}, x)$ $a_2 : (\{\neg y\}, \neg y)$

$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $a_4 : (\{x, \neg y\}, x \wedge \neg y)$

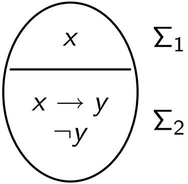
$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $a_6 : (\{x, x \rightarrow y\}, y)$



$a_1 > a_2, a_3, a_4, a_5, a_6$

Calculating pref-stable extensions: Example

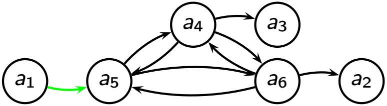
- An argument is a minimal proof
- $a\mathcal{R}b$ iff the conclusion of a contradicts a formula in the support of b
- $\geq$ = the weakest link principle



$a_1 : (\{x\}, x)$ $\quad$ $a_2 : (\{\neg y\}, \neg y)$

$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $\quad$ $a_4 : (\{x, \neg y\}, x \wedge \neg y)$

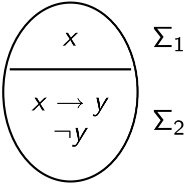
$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $\quad$ $a_6 : (\{x, x \rightarrow y\}, y)$



$a_1 > a_2, a_3, a_4, a_5, a_6$

Calculating pref-stable extensions: Example

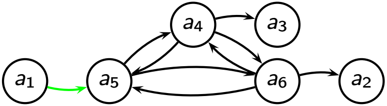
- An argument is a minimal proof
- $a\mathcal{R}b$ iff the conclusion of a contradicts a formula in the support of b
- $\geq$ = the weakest link principle



$a_1 : (\{x\}, x)$ $\quad a_2 : (\{\neg y\}, \neg y)$

$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $\quad a_4 : (\{x, \neg y\}, x \wedge \neg y)$

$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $\quad a_6 : (\{x, x \rightarrow y\}, y)$



Two pref-stable extensions:

$\mathcal{E}_1 = \{a_1, a_2, a_4\}$

$\mathcal{E}_2 = \{a_1, a_3, a_6\}$

$a_1 > a_2, a_3, a_4, a_5, a_6$

Preferred sub-theories (Brewka'89)

Let $\Sigma = \Sigma_1 \cup \ldots \cup \Sigma_n$ be a stratified propositional knowledge base.

Preferred sub-theories (Brewka'89)

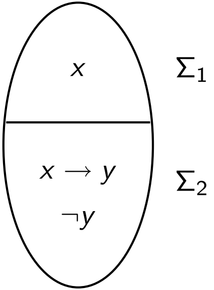
Let $\Sigma = \Sigma_1 \cup \ldots \cup \Sigma_n$ be a stratified propositional knowledge base.

Definition

Let $\mathcal{S} \subseteq \Sigma$ and $\mathcal{S}_i = \mathcal{S} \cap \Sigma_i$. $\mathcal{S}$ is a preferred sub-theory iff for every $1 \leq k \leq n$, $\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_k$ is a maximal consistent set in $\Sigma_1 \cup \ldots \cup \Sigma_k$

Preferred sub-theories (Brewka'89)

Let $\Sigma = \Sigma_1 \cup \ldots \cup \Sigma_n$ be a stratified propositional knowledge base.



Definition

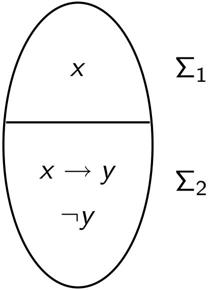
Let $\mathcal{S} \subseteq \Sigma$ and $\mathcal{S}_i = \mathcal{S} \cap \Sigma_i$. $\mathcal{S}$ is a preferred sub-theory iff for every $1 \leq k \leq n$, $\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_k$ is a maximal consistent set in $\Sigma_1 \cup \ldots \cup \Sigma_k$

Preferred sub-theories (Brewka'89)

Let $\Sigma = \Sigma_1 \cup \ldots \cup \Sigma_n$ be a stratified propositional knowledge base.

Definition

Let $\mathcal{S} \subseteq \Sigma$ and $\mathcal{S}_i = \mathcal{S} \cap \Sigma_i$.
$\mathcal{S}$ is a preferred sub-theory iff for every $1 \leq k \leq n$, $\mathcal{S}_1 \cup \ldots \cup \mathcal{S}_k$ is a maximal consistent set in $\Sigma_1 \cup \ldots \cup \Sigma_k$



$\mathcal{S}_1 = \{x, x \rightarrow y\}$
$\mathcal{S}_2 = \{x, \neg y\}$

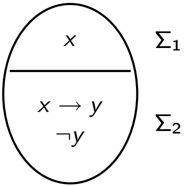
Recovering preferred sub-theories (Amgoud and Vesic, SUM'10)

- Let an argument be defined as a minimal proof in the form $(\texttt{Supp}, \texttt{Conc})$
- For $\mathcal{S} \subseteq \Sigma$, we define $\texttt{Arg}(\mathcal{S})$ as the set of all arguments that can be built from $\mathcal{S}$, i.e. all arguments a s.t. $\texttt{Supp}(a) \subseteq \mathcal{S}$.

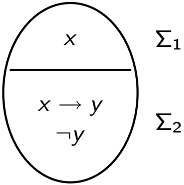
Theorem (7)

The pref-stable extensions of $\mathcal{T} = (\texttt{Arg}(\Sigma), Undercut, \geq_{wlp})$ are exactly the $\texttt{Arg}(\mathcal{S})$ where $\mathcal{S}$ ranges over the preferred sub-theories of Σ.

Recovering preferred sub-theories (Amgoud and Vesic, SUM'10)

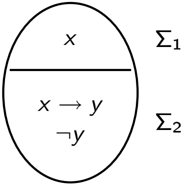


Recovering preferred sub-theories (Amgoud and Vesic, SUM'10)



$\mathcal{S}_1 = \{x, x \rightarrow y\}$
$\mathcal{S}_2 = \{x, \neg y\}$

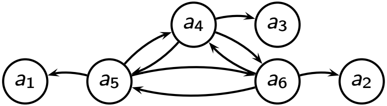
Recovering preferred sub-theories (Amgoud and Vesic, SUM'10)



$a_1 : (\{x\}, x)$ $\quad$ $a_2 : (\{\neg y\}, \neg y)$

$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $\quad$ $a_4 : (\{x, \neg y\}, x \wedge \neg y)$

$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $\quad$ $a_6 : (\{x, x \rightarrow y\}, y)$

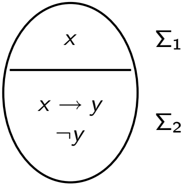


$a_1 > a_2, a_3, a_4, a_5, a_6$

$\mathcal{S}_1 = \{x, x \rightarrow y\}$

$\mathcal{S}_2 = \{x, \neg y\}$

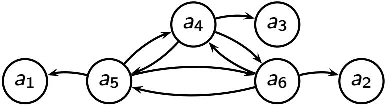
Recovering preferred sub-theories (Amgoud and Vesic, SUM'10)



$a_1 : (\{x\}, x)$ $\quad a_2 : (\{\neg y\}, \neg y)$

$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $\quad a_4 : (\{x, \neg y\}, x \wedge \neg y)$

$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $\quad a_6 : (\{x, x \rightarrow y\}, y)$



$a_1 > a_2, a_3, a_4, a_5, a_6$

$\mathcal{S}_1 = \{x, x \rightarrow y\}$

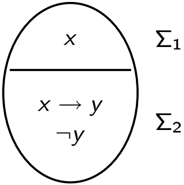
$\mathcal{S}_2 = \{x, \neg y\}$

$\mathcal{E}_1 = \texttt{Arg}(\mathcal{S}_1) = \{a_1, a_3, a_6, \ldots\}$

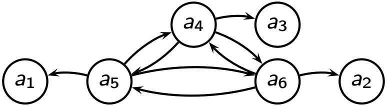
$\mathcal{E}_2 = \texttt{Arg}(\mathcal{S}_2) = \{a_1, a_2, a_4, \ldots\}$

Recovering preferred sub-theories (Amgoud and Vesic, SUM'10)

For every extension: no conflicting arguments, no contradictory conclusions, the base is consistent



$a_1 : (\{x\}, x)$ $\quad$ $a_2 : (\{\neg y\}, \neg y)$
$a_3 : (\{x \rightarrow y\}, x \rightarrow y)$ $\quad$ $a_4 : (\{x, \neg y\}, x \wedge \neg y)$
$a_5 : (\{\neg y, x \rightarrow y\}, \neg x)$ $\quad$ $a_6 : (\{x, x \rightarrow y\}, y)$



$a_1 > a_2, a_3, a_4, a_5, a_6$

$\mathcal{S}_1 = \{x, x \rightarrow y\}$
$\mathcal{S}_2 = \{x, \neg y\}$

$\mathcal{E}_1 = \texttt{Arg}(\mathcal{S}_1) = \{a_1, a_3, a_6, \ldots\}$
$\mathcal{E}_2 = \texttt{Arg}(\mathcal{S}_2) = \{a_1, a_2, a_4, \ldots\}$

Conclusion

- A new approach for PAF that palliates the limits of existing ones

Conclusion

- A new approach for PAF that palliates the limits of existing ones
- Semantics defined in terms of dominance relations that compare any pair of subsets of arguments

Conclusion

- A new approach for PAF that palliates the limits of existing ones
- Semantics defined in terms of dominance relations that compare any pair of subsets of arguments
- The new semantics are defined in terms of postulates

Conclusion

- A new approach for PAF that palliates the limits of existing ones
- Semantics defined in terms of dominance relations that compare any pair of subsets of arguments
- The new semantics are defined in terms of postulates
- Dung's semantics expressed in the new setting

Conclusion

- A new approach for PAF that palliates the limits of existing ones
- Semantics defined in terms of dominance relations that compare any pair of subsets of arguments
- The new semantics are defined in terms of postulates
- Dung's semantics expressed in the new setting
- The approach recovers well-known works on handling inconsistency (preferred sub-theories, demo-preferred sub-theories)

Questions

- Thank you
- Questions