Dialectical Proofs for Constrained Argumentation

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Overview

1. Argumentation Frameworks
   - Dung's Argumentation Framework
   - Constrained Argumentation Framework
2. Constrained dialectical proofs
   - Dialectical framework
   - Definition of constrained dialectical proofs
   - Computation
3. Conclusion

Argumentation framework - Definition

- [Dung95] An argumentation framework is a pair $\langle A, R \rangle$ where:
  - $\mathcal{A}$ is a set of arguments
  - $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ represents a notion of attack
- Can be represented as a directed graph

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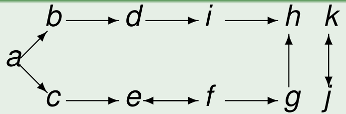

Argumentation framework - Semantics

- A subset $S \subseteq \mathcal{A}$ is admissible if:
  1. S is conflict-free: there are not two arguments in S such that one attacks the other, and
  2. S defends all its elements: any argument $y \in \mathcal{A} \setminus S$ that attacks $x \in S$ is attacked by some $z \in S$.
- S is a preferred extension iff it is maximal w.r.t. $\subseteq$ among the set of admissible sets.

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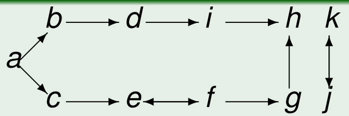

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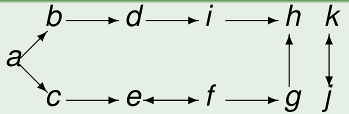

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- S is a preferred $\mathcal{C}$-extension iff it is maximal w.r.t. $\subseteq$ among the set of $\mathcal{C}$-admissible sets.
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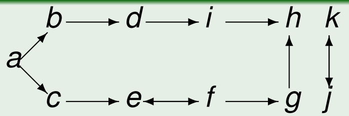

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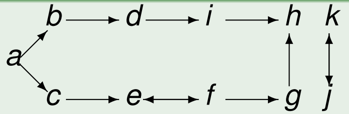

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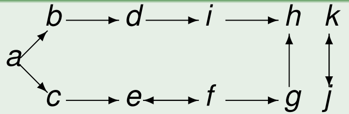

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Constrained argumentation framework - Properties

- Generalizes other argumentation frameworks and semantics [Coste-Marquis *et al.* 06], e.g.:
  - Dung's argumentation framework and the preferred semantics
    - Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. Let $CAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ be a constrained argumentation framework where $\mathcal{C}$ is any valid formula. Then the preferred extensions of AF are the preferred $\mathcal{C}$-extensions of CAF.
  - Cayrol and Lagasquie-Schiex's bipolar argumentation framework and the weakly c-preferred semantics
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Credulous acceptance problem

- Credulous acceptance problem under the $\mathcal{C}$-preferred semantics:

Given a CAF $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$,
is a given set $S \subseteq \mathcal{A}$
included in at least one preferred $\mathcal{C}$-extension of CAF?

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$\mathcal{C} = (k \Leftrightarrow d) \wedge ((d \Rightarrow (f \vee g)) \vee (\neg d \Rightarrow \neg f))$

Is $\{e, k\}$ included in at least one preferred $\mathcal{C}$-extension?

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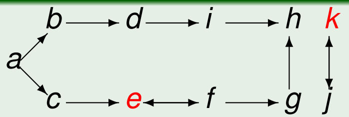

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Dialectical framework

Adaptation of [Cayrol *et al.* 03] definitions:

Definition

Let $\mathcal{A}$ be a set of arguments. Let _ be an "empty" argument.
A dialogue is a finite sequence

$$d = \langle a_0.a_1.a_2 \ldots a_n \rangle$$

of arguments from $\mathcal{A}^- = \mathcal{A} \cup \{\_\}$.
The player of a_i, $i \in \{0 \ldots n\}$, in d is:

- PRO if i is even and
- OPP if i is odd

Dialectical framework

Definition

Let $\phi : \mathcal{A}^{-*} \to 2^{\mathcal{A}^-}$ a function called legal-move function.
A ϕ-dialogue for a set of arguments $S \subseteq \mathcal{A}$ is a dialogue d such that:

1. $\forall i \geq 0, a_i \in \phi(d_i)$, and
2. S is included in PRO(d), the set of arguments played by PRO in d.

Constrained dialectical proofs

$\Rightarrow$ Specific legal-move function $\phi_{\mathcal{C}}$ defined to answer the credulous acceptance problem under the $\mathcal{C}$-preferred semantics.

Definition

Let $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ be a constrained argumentation framework and $S \subseteq \mathcal{A}$ be a set of arguments.

A $\phi_{\mathcal{C}}$-proof for S is a $\phi_{\mathcal{C}}$-dialogue d for S such that:

1. either d is empty or d ends with the empty argument, and
2. the set of arguments played by PRO in d satisfies $\mathcal{C}$

We say that d is won by PRO.

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Proposition (Correctness and Completeness)

Let CAF $= \langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ *be a constrained argumentation framework.*

- *If d is a* $\phi_{\mathcal{C}}$*-proof for a set of arguments S* $\subseteq \mathcal{A}$*, then the set of arguments played by PRO in d is a* $\mathcal{C}$*-admissible set of CAF that contains S.*
- *If a set of arguments S is included in a* $\mathcal{C}$*-admissible set of CAF then there exists a* $\phi_{\mathcal{C}}$*-proof for S.*

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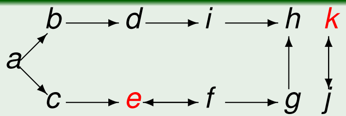

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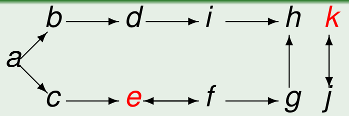

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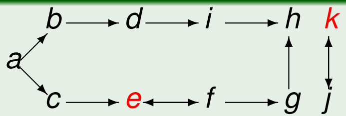

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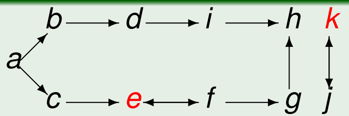

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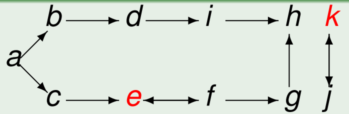

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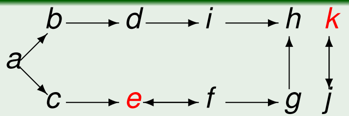

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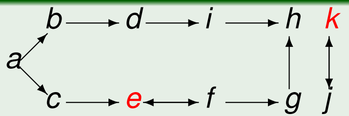

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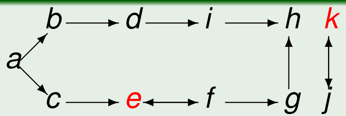

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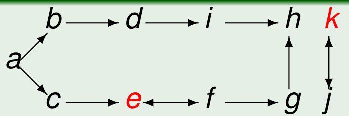

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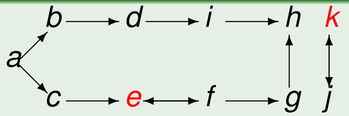

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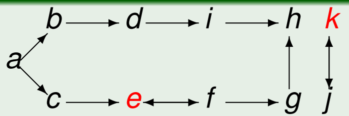

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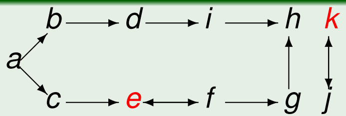

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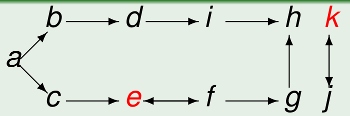

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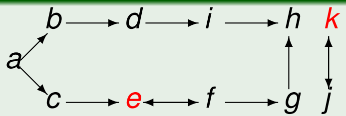

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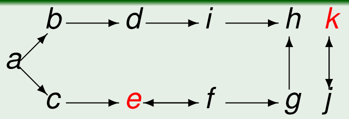

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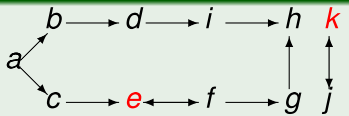

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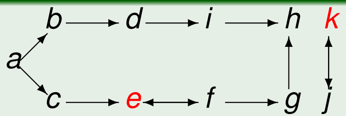

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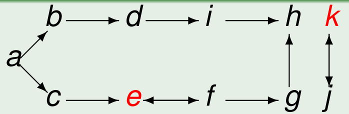

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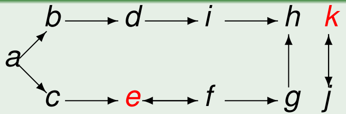

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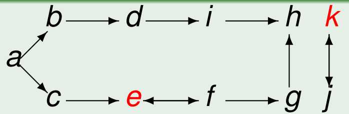

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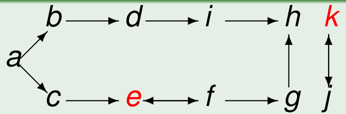

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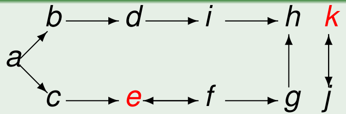

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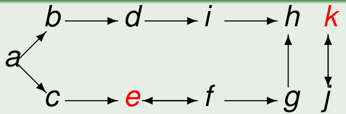

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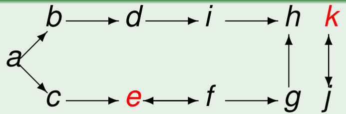

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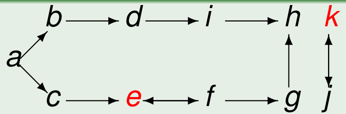

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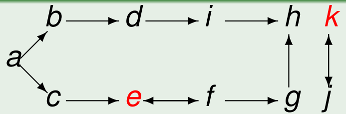

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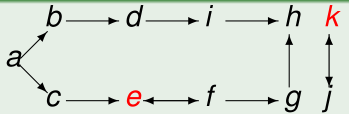

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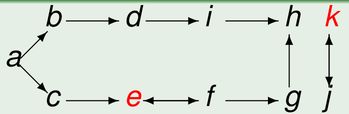

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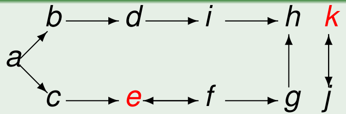

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Constrained dialectical proofs - Example

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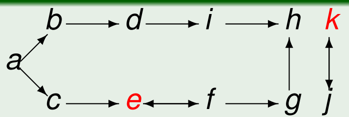

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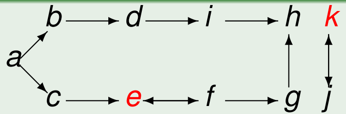

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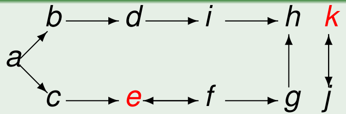

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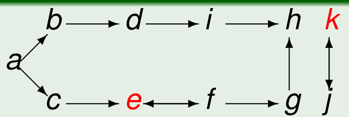

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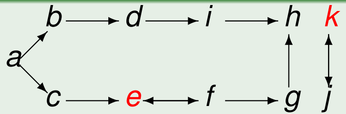

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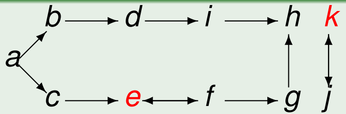

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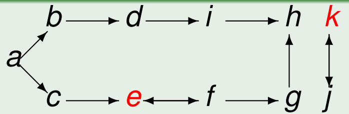

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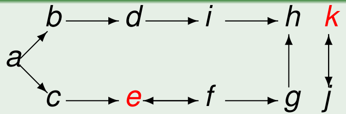

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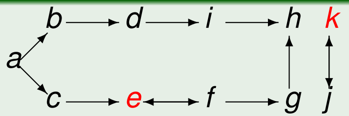

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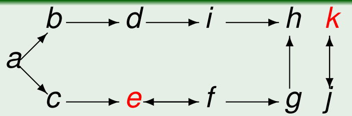

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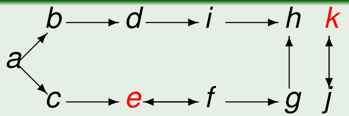

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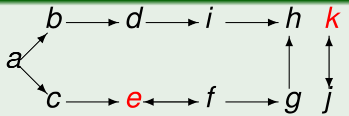

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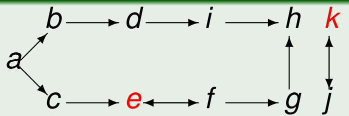

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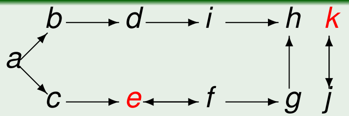

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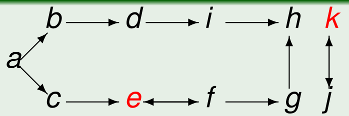

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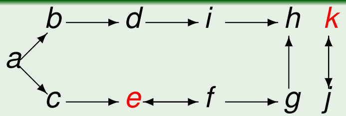

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Computation by ASP

- Answer Set Programming:
  - Simple and readable encoding
  - Well adapted to encode the iterating and alternating roles of pros and cons
- Computation of the constrained dialectical proofs:
  - In the ASP solver `ASPeRiX`
  - Program available at `http://www.info.univ-angers.fr/pub/claire/asperix/Argumentation`

Conclusion

- Constrained argumentation frameworks: generalize other existing frameworks and semantics
- Simple and general dialectical framework
- Dialectical proofs for the credulous acceptance problem under the $\mathcal{C}$-preferred semantics
- Computation in ASP