On Extension Counting Problems in Argumentation Frameworks

Pietro Baroni, Paul E. Dunne & Massimiliano Giacomin

Univ. of Brescia, Italy &

Univ. of Liverpool, UK

Overview

- Decision, Listing and Counting
- Motivating counting problems.
- Resolution-based semantics.
- General intractability results.
- Some tractable cases.
- Summary.

Decision, Listing & Counting

- The majority of algorithmic study on extension-based semantics in AFs has focused on two classes of computation:
- *Decision problems* (existence, verification, acceptability status, etc)
- *Enumeration* (listing) problems (construct all extensions of a particular type).
- *Counting* problems have tended to be neglected (except in so far as counting is a by-product of enumeration).

Why counting?

- Some natural questions about argument status are motivated in terms of counting problems, e.g. "does x occur in *some fraction* ε of all extensions?".
- Counting can give some guidance on how well "*average case*" approaches perform, e.g. if there are $f(n)$ extensions of a given type then $f(n)/2^n$ indicates how likely a *randomly chosen* subset is to be a suitable extension.

Resolution-based semantics I

- Parametric approach introduced by Baroni & Giacomin (COMMA, 2008).
- Start with the set of "*mutual attacks*" in $<X,A>$, i.e. the set
$$M=\{\{p,q\} : <p,q>\in A, <q,p>\in A\}$$
- A (full) resolution, β of $<X,A>$ contains *exactly one* attack from A for each $\{p,q\}$ in M.
- The *resolution-based* σ-*extensions* are the *minimal* sets in $E_\sigma(<X,A\backslash\beta>)$ over all full resolutions.

Resolution-based semantics II

- The resolution-based grounded semantics – GR* – is a *multiple-status* semantics (grounded semantics – GR – in AFs being *unique status*) shown to have many useful properties (Baroni & Giacomin, 2008).
- Given <X,A> & $S \subseteq X$, verifying if S is in the set of GR* extensions of <X,A> is *polynomial time* decidable.
- Determining if {GR(<X,A>)}=GR*(<X,A>) is *polynomial time* decidable.
- Proofs given in Baroni et al. (IJCAI, 2009).

Summary of Results

- Denote by Count$_\sigma$(<X,A>) the *function* which reports the number of subsets S of X s.t. S is a σ-extension in <X,A>.
- If $\sigma \in$ {ST,PR,Naïve} and <X,A> is *symmetric* then Count$_\sigma$(<X,A>) is #P-complete.
- If σ=GR* and <X,A> is *symmetric* then Count$_\sigma$(<X,A>) is in FP.
- If $\sigma \in$ {ST,PR,GR*} and <X,A> is "*tree-like*" then Count$_\sigma$(<X,A>) is in FP.

Symmetry and SCC-symmetry

- An AF <X,A> is *symmetric* if
  $\forall x,y \in X \; \langle x,y \rangle \in A \Leftrightarrow \langle y,x \rangle \in A$
- An AF is *SCC-symmetric* if every component of its *strongly connected component* (SCC) decomposition is symmetric.
- SCC-symmetric AFs are *coherent*. (every preferred extension is *stable*)
- *Maximal conflict-free* (MCF) sets (i.e. naïve extensions) *coincide with preferred extensions* in *symmetric* AFs.

“Tree-like” AFs

- We say an (SCC-symmetric) AF is *tree-like* if its *undirected* structure is *acyclic* and its SCC decomposition has *exactly one maximal element*, i.e. there is a unique component, C_{max}, for which $\forall x \in C_{max}\ \forall y \notin C_{max}$ there is no directed path from x to y in <X,A>.
- If <X,A> is tree-like then GR*, ST and PR extensions coincide.

“Negative” results I

- The (function) complexity class #P corresponds to problems equivalent to *counting* the number of *accepting computations* of a non-deterministic TM.
- If L is an NP decision problem, then #L, the problem of counting the number of witnesses to an instance, x, being in L is in #P, e.g. counting the number of distinct satisfying assignments of $\varphi(Z_n)$ is in #P.
- If L is NP-complete then #L is #P-complete.
- Many cases are known where *deciding* $x \in L$ is “*easy*” but *counting the number of solutions* is #P-complete, e.g. perfect matchings in bipartite graphs.

Negative results II

A. For $\sigma \in \{ST, PR, Naïve\}$ & <X,A> *symmetric*, $Count_{\sigma}(<X,A>)$ is #P-complete: immediate from fact that counting the number of MCF sets is #P-complete. (Valiant, 1979)

B. $Count_{GR^*}$ is #P-complete: does *not* follow from (A) (recall minimality condition) but is shown by constructing an AF from a CNF, $\varphi(Z)$, in which the number of resolution-based grounded sets is exactly

$$2^n + |\{\alpha : \varphi(\alpha)\}|$$

Positive results

- From (Baroni, Dunne & Giacomin, 2009) we know that (for *symmetric* AFs)

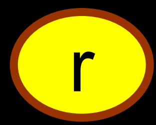
  $$\mathrm{Count}_{GR^*}(\langle X,A\rangle)>1$$

  $$\Leftrightarrow$$

  <X,A> has no cycles of length >2 & |X|>1
- Thus the *only* case to be considered is if the *undirected graph* form is *acyclic*, i.e. a *tree*.
- The efficient algorithm for this case is a sub-routine for counting in SCC-symmetric AFs.

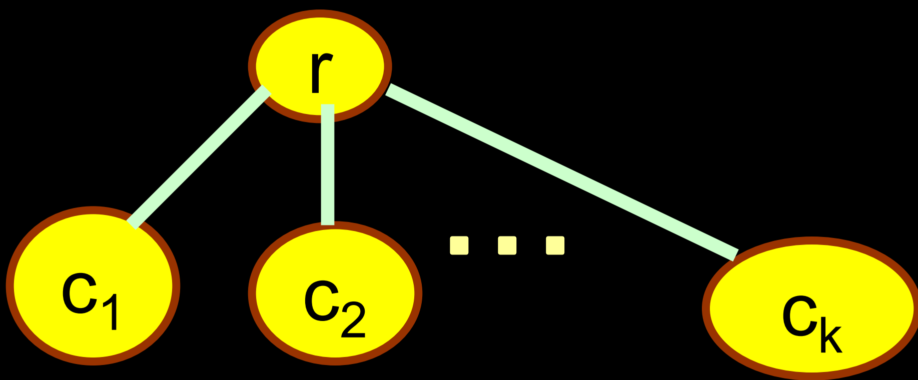
Basic approach – symmetric AFs

- Exploit correspondence between MCF sets and GR* extensions for the sole "non-trivial" case.
- This uses the dynamic programming method presented in (Wilf, 1984).
- In this, a relationship between #MCFs contributed by sub-trees is derived.



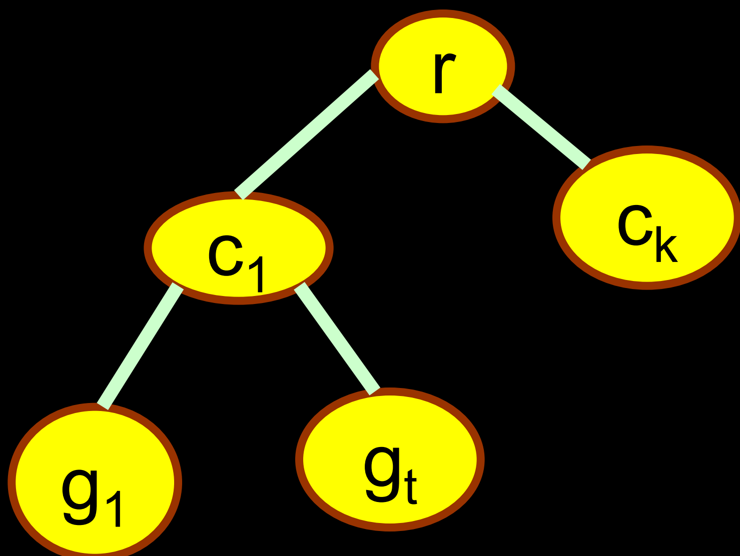
#MCF = 1

not containing r=0



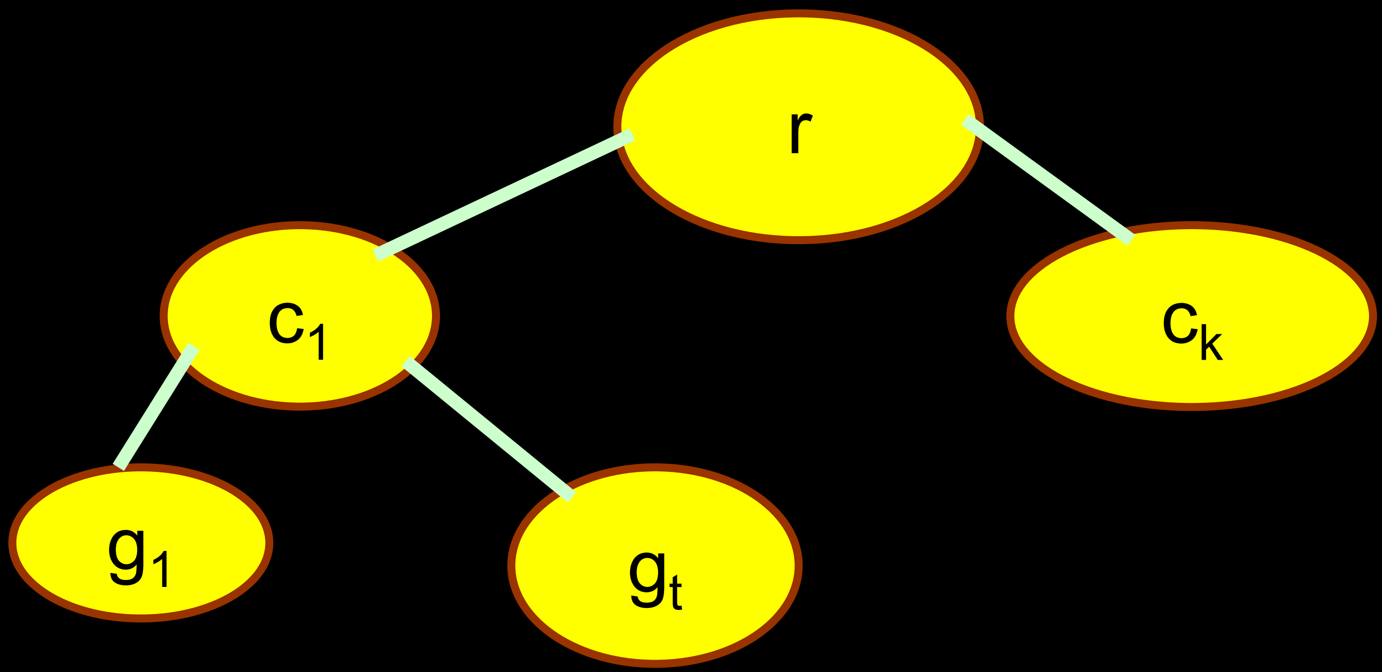
#MCF =2

not containing r=1



#MCF = #(not containing r)+

Π_g (#MCF sub-tree root g)

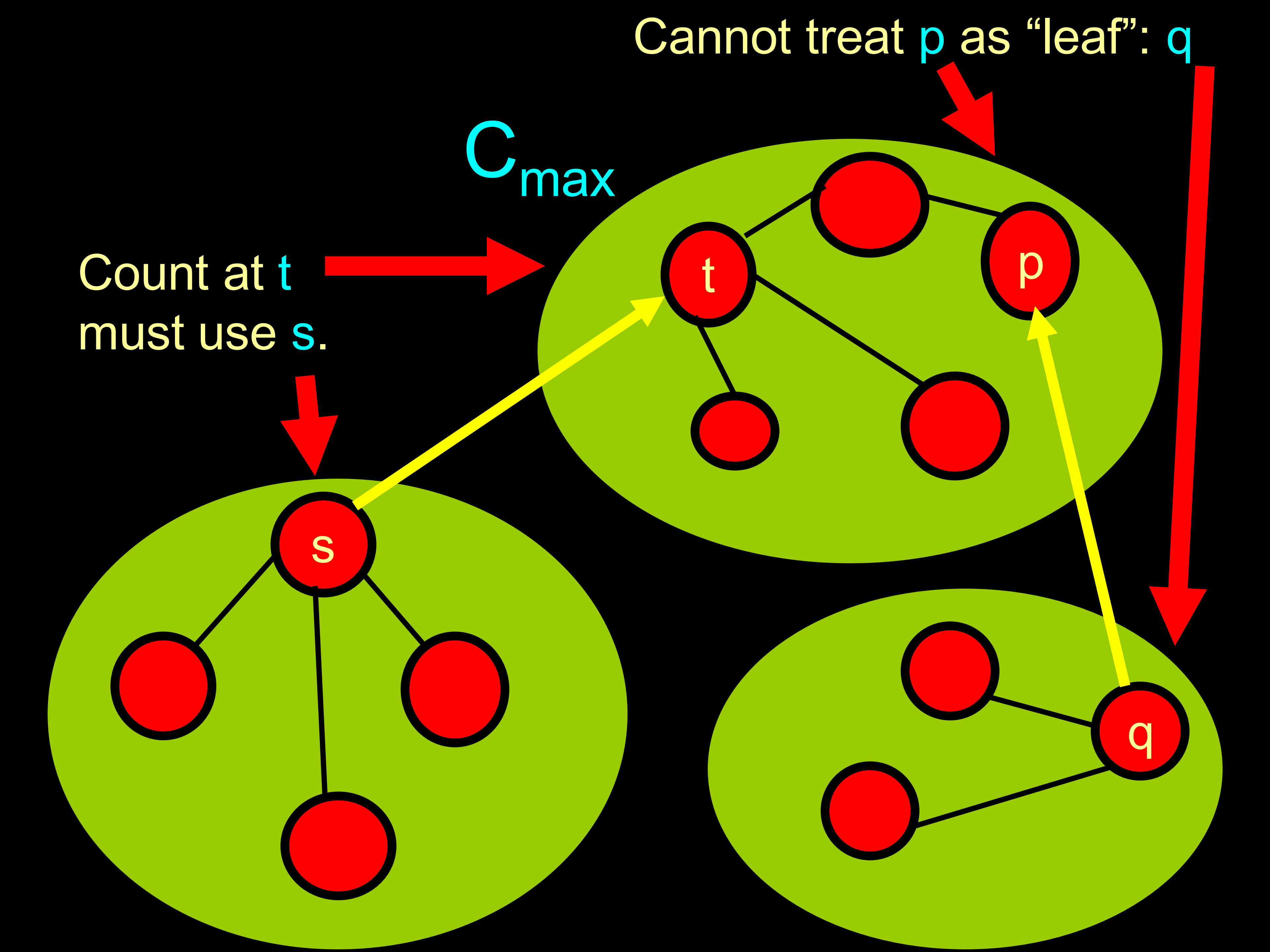


#MCF(not containing r) =

Π_c (#MCF(in sub-tree at c) –
Π_c (#MCF(at c not containing c)

Tree-like AFs

- Can use essentially same approach to compute Count_{GR^*} in tree-like AFs, but since these are not, in general, symmetric a correction is needed.
- Problem is that the (unique) attack linking *one component to another* is not a *mutual attack*.


Cannot treat p as "leaf": q
C_{max}
Count at t
must use s.
t
p
s
q

More on tree-like AFs

- The correction needed to get accurate counts relies on the fact that the SCC decomposition has a unique maximal component: hence, every (non-maximal component) has exactly one immediate successor.
- This fails if more than 1 maximal component.
- Although polynomial time methods still apply, naïve implementations increase run-time exponentially with the number of maxima.

Conclusions & Development

- Some scope for efficient counting algorithms in extension-based semantics of AFs.
- Of interest to consider alternative (multiple status!) semantics and tractable graph cases.
- In principle, variants of Courcelle's Thm. can be adapted to count structures so that bounded treewidth AFs may be tractable w.r.t. some counting problems.