Base Logics in Argumentation

Anthony Hunter

Dept of Computer Science, UCL, London
Base logics for argumentation

In general, we can regard a logical argument as a tuple $\langle \Phi, \alpha \rangle$ where

- $\Phi$ is the support
- $\alpha$ is the claim
- $\Phi$ entails $\alpha$

In addition, we may require that $\Phi$ is consistent and/or $\Phi$ is minimal for entailing $\alpha$.

So arguments are constructed using some base logic $\vdash_x$ such as classical logic, defeasible logic, contrapositive logic, temporal logic, description logic, etc.
Classical logic for argumentation

Let $\Delta$ be set of formulas in classical logic

An argument is a pair $\langle \Phi, \alpha \rangle$ such that

1. $\Phi \not\vdash \bot$
2. $\Phi \vdash \alpha$
3. $\Phi$ is a minimal subset of $\Delta$ satisfying 2

For $\Delta = \{\neg\neg a, \neg b \rightarrow \neg a, \neg b \lor (c \land d), b \land c \land \neg b, \neg f \rightarrow g \lor h\}$, the following is an argument

$\langle \{\neg\neg a, \neg b \rightarrow \neg a, \neg b \lor (c \land d)\}, e \rightarrow d \rangle$
Defeasible logics for argumentation

Let $\Delta$ be the union of a set of rules and a set of literals. The defeasible logic consequence relation $\vdash_d$ is defined as follows.

$$\Delta \vdash_d \psi \text{ iff there is a sequence of literals } \alpha_1, \ldots, \alpha_n$$

such that $\psi$ is $\alpha_n$ and for each $\alpha_i \in \{\alpha_1, \ldots, \alpha_n\}$

either $\alpha_i$ is a literal in $\Delta$

or there is a $\beta_1 \land \ldots \land \beta_j \rightarrow_k \alpha_i \in \Delta$

and $\{\beta_1, \ldots, \beta_j\} \subseteq \{\alpha_1, \ldots, \alpha_{i-1}\}$

Let $\Delta = \{p, \neg q, p \rightarrow_1 \neg r, \neg q \land \neg r \rightarrow_2 s, s \rightarrow_1 t, p \land t \rightarrow_2 u\}$. Therefore $\Delta \vdash_d u$ where the derivation is $p, \neg r, \neg q, s, t, u$. 
Defeasible logics for argumentation

For $\Delta = \{p, \neg q, p \rightarrow_1 \neg r, \neg q \land \neg r \rightarrow_2 s, s \rightarrow_1 t, p \land t \rightarrow_2 u\}$, the following is an argument in defeasible logic programming.

$$\langle \{p, \neg q, p \rightarrow_1 \neg r, \neg q \land \neg r \rightarrow_2 s, s \rightarrow_1 t, p \land t \rightarrow_2 u\}, u \rangle$$

For $\Delta = \{p, \neg q, s, p \rightarrow \neg r, \neg q \land \neg r \land s \rightarrow t, t \land p \rightarrow u, v\}$, the following is an argument in assumption-based argumentation.

$$\langle \{p, \neg q, s, p \rightarrow \neg r, \neg q \land \neg r \land s \rightarrow t\}, t \rangle$$

The are limitations of defeasible logic. Consider $\Delta = \{a \rightarrow b, \neg a \rightarrow b\}$. Hence, $\Delta \vdash_c b$, but $\Delta \not\vdash_d b$. 
Defeasible logics for argumentation

Let \( \Delta \) be the union of a set of enhanced defeasible rules and a set of strong literals. The enhanced consequence relation \( \vdash_e \) is defined as follows.

\[
\Delta \vdash_e \psi \text{ iff there is a sequence of literals } \alpha_1, \ldots, \alpha_n \\
such that \psi \text{ is } \alpha_n \text{ and for each } \alpha_i \in \{\alpha_1, \ldots, \alpha_n\} \\
either \alpha_i \text{ is a strong literal in } \Delta \\
or there is a } \gamma_0 \land \ldots \land \gamma_m \land \beta_0 \land \ldots \land \beta_n \rightarrow_k \delta \in \Delta \\
and \{\gamma_0, \ldots, \gamma_m\} \subseteq \{\alpha_1, \ldots, \alpha_{i-1}\}
\]

Let \( \Delta = \{p, p \rightarrow_1 \neg r, \neg r \land \neg q \rightarrow_2 s, s \rightarrow_1 t, p \land t \rightarrow_1 u\} \).
Therefore \( \Delta \vdash_d u \) where the derivation is \( p, \neg r, s, t, u \).

\[
\langle \{p, p \rightarrow_1 \neg r, \neg r \land \neg q \rightarrow_2 s, s \rightarrow_1 t, p \land t \rightarrow_1 u\}, u \rangle
\]
Contrapositive logic for argumentation

Let $\Delta$ be a set of rules and literals. The defeasible logic consequence relation $\vdash_f$ is defined as follows.

$$\Delta \vdash_f \psi \text{ iff there is a sequence of literals } \alpha_1, \ldots, \alpha_n$$

such that $\psi$ is $\alpha_n$ and for each $\alpha_i \in \{\alpha_1, \ldots, \alpha_n\}$

either $\alpha_i$ is a literal in $\Delta$

or there is a $\beta_1 \land \ldots \land \beta_j \rightarrow_k \alpha_i \in \Delta \cup \text{Contrapositives}(\Delta)$

and $\{\beta_1, \ldots, \beta_j\} \subseteq \{\alpha_1, \ldots, \alpha_{i-1}\}$

Let $\Delta = \{q, \neg r, p \land q \rightarrow r, \neg p \rightarrow u\}$.

So $\text{Contrapositives}(\Delta) = \{\neg r \land q \rightarrow \neg p, p \land \neg r \rightarrow \neg q, \neg u \rightarrow p\}$.

Therefore, $\Delta \vdash_f u$, where the derivation is $q, \neg r, \neg p, u$. 
Metatheorems of the consequence relation

\( \Delta \cup \{\alpha\} \vdash x \alpha \)
(Reflexivity)

\( \Delta \cup \{\alpha\} \vdash x \alpha \) if \( \alpha \) is a literal
(Literal reflexivity)

\( \Delta \cup \{\beta\} \vdash x \gamma \) if \( \Delta \cup \{\alpha\} \vdash x \gamma \) and \( \vdash \alpha \leftrightarrow \beta \)
(Left logical equivalent)

\( \Delta \vdash x \alpha \) if \( \Delta \vdash x \beta \) and \( \vdash \beta \rightarrow \alpha \)
(Right weakening)

\( \Delta \vdash x \alpha \land \beta \) if \( \Delta \vdash x \alpha \) and \( \Delta \vdash x \beta \)
(And)

\( \Delta \cup \{\alpha\} \vdash x \beta \) if \( \Delta \vdash x \beta \)
(Monotonicity)

\( \Delta \vdash x \beta \) if \( \Delta \vdash x \alpha \) and \( \Delta \cup \{\alpha\} \vdash x \beta \)
(Cut)

\( \Delta \vdash x \alpha \rightarrow \beta \) if \( \Delta \cup \{\alpha\} \vdash x \beta \)
(Conditionalization)

\( \Delta \cup \{\alpha\} \vdash x \beta \) if \( \Delta \vdash x \alpha \rightarrow \beta \)
(Deduction)

\( \Delta \cup \{\alpha\} \vdash x \beta \) if \( \Delta \cup \{\neg \beta\} \vdash x \neg \alpha \)
(Contraposition)

\( \Delta \cup \{\alpha \lor \beta\} \vdash x \gamma \) if \( \Delta \cup \{\alpha\} \vdash x \gamma \) and \( \Delta \cup \{\beta\} \vdash x \gamma \)
(Or)
Inconsistency tolerance

- The consequence relation $\vdash_x$ is **trivializable** iff for all $\Delta$, there is an atom $\alpha$ such that if $\Delta \vdash \alpha$ and $\Delta \vdash \neg \alpha$ then $\Delta \vdash_x \beta$ for all atoms $\beta$.

- A formula $\alpha$ is pure w.r.t. $\Delta$ iff $\text{Atoms}(\Delta) \cap \text{Atoms}(\{\alpha\}) \neq \emptyset$.
  A consequence relation $\vdash_x$ is **pure** iff for all $\alpha$ and $\Delta$, if $\Delta \vdash_x \alpha$, then $\alpha$ is pure w.r.t. $\Delta$.

Classical logic is trivializable and not pure, whereas defeasible logics and contrapositive logic are pure and not trivializable.

Even though most definitions for a logical argument assume consistency of premises, we may wish to allow paraconsistent inferences from inconsistent premises.
### Decision problems

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<thead>
<tr>
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<th>Entailment</th>
<th>Consistency checking</th>
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<tr>
<td>Classical</td>
<td>coNPC</td>
<td>NPC</td>
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<tr>
<td>Contrapositive</td>
<td>coNPC</td>
<td>NPC</td>
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<tr>
<td>Defeasible</td>
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<td>P</td>
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For defeasible logic, there are polynomial time algorithms (see Mahler TPLP 2004)
Comparing base logics

- Meta-theorems of the consequence relation
  
  defeasible $<$ contrapositive $<$ classical

- Inferential strength of the consequence relation
  
  defeasible $<$ contrapositive $<$ classical

- Paraconsistent properties of the consequence relation
  
  defeasible & contrapositive are tolerant, whereas classical is not tolerant

- Complexity of entailment and consistency decision problems
  
  defeasible is tractable, whereas contrapositive & classical are not tractable

Each base logic has strengths and weaknesses for applications.
Combined logics for argumentation

Some formalisms can be viewed as a composition of logics (called bilogics).

- Some defeasible logic formalisms (e.g. defeasible logic programming, extended logic programming, etc) which involve multiple implication symbols such as for “defeasible” and for “strict” rules.

- Some hybrid formalisms for using defeasible logic with ontologies (e.g. a variant of defeasible logic programming in which conditions of defeasible rules can be evaluated by sub-contract to an ontology).
Combined logics for argumentation

Some logic-based approaches involve a combination of logics ($\vdash_1, \vdash_2, ...$) for the definition of entailment ($\vdash$) when generating arguments.

$$(\Delta_1, \Delta_2) \vdash \alpha \iff$$

$$(\Delta_1, \Delta_2) \vdash \beta_1 \text{ and } \ldots (\Delta_1, \Delta_2) \vdash \beta_n$$

and $$(\Delta_1 \cup \{\beta_1, \ldots, \beta_n\} \vdash_1 \alpha \text{ or } \Delta_2 \cup \{\beta_1, \ldots, \beta_n\} \vdash_2 \alpha)$$

A combined logic allows for the use of specialised logics for specialized knowledge.

For example, in Defeasible Logic Programming (Garcia+Simari) and Extended Logic Programming (Prakken+Sartor) where $\Delta_1$ is a set of strict rules and $\Delta_2$ is a set of defeasible rules, and each of $\vdash_1$ and $\vdash_2$ are modus ponens).
Combined logics for argumentation

\[(\Delta_1, \Delta_2) \vdash \alpha \text{ iff } (\Delta_1, \Delta_2) \vdash \beta_1 \text{ and } \ldots (\Delta_1, \Delta_2) \vdash \beta_n \text{ and } (\Delta_1 \cup \{\beta_1, \ldots, \beta_n\} \vdash_1 \alpha \text{ or } \Delta_2 \cup \{\beta_1, \ldots, \beta_n\} \vdash_2 \alpha)\]

E.g. \(\Delta_1\) is strict and \(\Delta_2\) is defeasible.

\[\Delta_1 = \{a, a \rightarrow c, c \land b \rightarrow e\}\]

\[\Delta_2 = \{a \rightarrow b, e \rightarrow f\}\]

Hence

\[(\Delta_1, \Delta_2) \vdash f\]
Combined logics for argumentation

A disadvantage is that counter-intuitive reasoning may arise (as identified by Caminada+Amgoud) when using weak logics (e.g. logics without contraposition).

E.g. $\Delta_1$ is strict and $\Delta_2$ is defeasible.

$\Delta_1 = \{\text{weddingring, clubber, married} \rightarrow \text{spouse}, \text{single} \rightarrow \neg \text{spouse}\}$

$\Delta_2 = \{\text{weddingring} \Rightarrow \text{married}, \text{clubber} \Rightarrow \text{single}\}$

Hence

$(\Delta_1, \Delta_2) \vdash \text{married}$

$(\Delta_1, \Delta_2) \vdash \text{spouse}$

$(\Delta_1, \Delta_2) \vdash \text{single}$

$(\Delta_1, \Delta_2) \vdash \neg \text{spouse}$

Caminada+Amgoud suggest some interesting postulates for constraining argument systems (taking into account the definitions for entailment, attack and extensions).
Combined logics for argumentation

A more cautious approach to combining logics “ring-fences” the knowledgebase of one of the logics.

\[(\Delta_1, \Delta_2) \vdash \alpha \text{ iff } \Delta_1 \vdash_1 \alpha\]

or \[((\Delta_1, \Delta_2) \vdash \beta_1 \text{ and } \ldots (\Delta_1, \Delta_2) \vdash \beta_n \text{ and } \Delta_2 \cup \{\beta_1, \ldots, \beta_n\} \vdash_2 \alpha)\]

E.g. \(\Delta_1\) is strict and \(\Delta_2\) is defeasible.

\[\Delta_1 = \{\text{weddingring, clubber, married} \rightarrow \text{spouse}, \text{single} \rightarrow \neg \text{spouse}\}\]
\[\Delta_2 = \{\text{weddingring} \rightarrow \text{married}, \text{clubber} \rightarrow \text{single}\}\]

Hence

\[(\Delta_1, \Delta_2) \vdash \text{married} \quad (\Delta_1, \Delta_2) \not\vdash \text{spouse}\]
\[(\Delta_1, \Delta_2) \vdash \text{single} \quad (\Delta_1, \Delta_2) \not\vdash \neg \text{spouse}\]
Ontology-based argumentation

- $\Delta_1$ is a set of defeasible rules of the form $\beta_1 \land \ldots \beta_n \rightarrow \beta_{n+1}$ and $\vdash_1$ is modus ponens.
- $\Delta_2$ is a set of classical or description logic formulae and $\vdash_2$ is classical/description logic consequence relation.

$$(\Delta_1, \Delta_2) \vdash \alpha \text{ iff }$$
$$(\Delta_1, \Delta_2) \vdash \beta_1 \text{ and } \ldots (\Delta_1, \Delta_2) \vdash \beta_n$$
and
$$(\Delta_1 \cup \{\beta_1, \ldots, \beta_n\} \vdash_1 \alpha \text{ or } \Delta_2 \cup \{\beta_1, \ldots, \beta_n\} \vdash_2 \alpha)$$

E.g. $\Delta_1 = \{a \rightarrow b, e \rightarrow f\}$ and $\Delta_2 = \{a, \neg b \lor \neg c, \neg e \rightarrow c\}$.

$$(\Delta_1, \Delta_2) \vdash f$$
Ontology-based argumentation

Let $\Delta_1$ be set of defeasible rules and let $\Delta_2$ be an ontology.

An argument is a pair $\langle \Phi, \alpha \rangle$ such that

1. $\Phi$ is consistent
2. $\Phi$ is a minimal subset of $\Delta_1$ s.t. $(\Phi, \Delta_2) \vdash \alpha$

Example: Let $\Delta_1 = \{ b \land c \rightarrow d, d \rightarrow f, g \rightarrow \neg d, i \rightarrow \neg f \}$
and $\Delta_2 = \{ a, a \rightarrow b, c, h, h \rightarrow g, i \}$

$\langle \{ b \land c \rightarrow d, d \rightarrow f \}, f \rangle$
$\langle \{ g \rightarrow \neg d \}, \neg d \rangle$ (undercut)
$\langle \{ i \rightarrow \neg f \}, \neg f \rangle$ (rebut)
Ontology-based argumentation

Some of the content in the OWL ontology

CLASS NAMES

DrugRegimes
  Bisphosphonates
    ZoledronicAcid
HormoneRA
  Tamoxifen
    Tamoxifen5Yr
    Tamoxifen2Yr
Anastrazole
Diseases
  Breast Cancer
    ERPos Breast Cancer
    ERNeg Breast Cancer
EndometrialCancer

PROPERTIES

hasDisease(Person, Disease)
hasTreatment(Person, Treatment)
hasAge(Person, int)

INSTANCES

Tamoxifen5Yr(TamCourseA)
Tamoxifen2Yr(TamCourseB)
Anastrazole(AnastrazoleCourseA)
ZoledronicAcid(ZolendronateA)

Example from Williams & Hunter, ICTAI’07
Ontology-based argumentation

An example of a defeasible rule implying changed risk of breast cancer disease-free survival.

\[
\text{Women}(x) \\
\quad \land \text{hasDisease}(x,y) \land \text{EarlyBreastCancer}(y) \land \text{ERPositiveDisease}(y) \\
\quad \land \text{hasTreatment}(x,z) \land \text{Tamoxifen5Yr}(z) \\
\quad \land \text{IncreasedBrCaDFS}(a) \\
\Rightarrow \text{hasDeltaRisk}(x,a)
\]

Example from Williams & Hunter, ICTAI’07
Ontology-based argumentation

- **Exploit the ontology as a repository of strict knowledge**: Use instances in ontology (ABox) as facts, and use class + relationship definitions in ontology (TBox) to infer further facts.

- **Exploit rigour of the ontology for the rule language**: Use class + relationship names of ontology as the predicate symbols in rules.

- **Exploit ontology reasoning software**: Evaluate each condition in a rule by querying the ontology.

- **Exploit argumentation system**: For generating and comparing arguments (e.g. DeLP, ASPIC, CASAPI, etc).

- **Exploit the ontology to determine conflict between pairs of atoms**: Conflict is redefined in terms of inconsistency in the ontology.
Conclusions

- Most logical argument systems are based on a simple defeasible logic, but there are many other base logic available in KR and CS.
- There are a number of ways that we can compare base logics
- Choice of base logic is important in designing a logical argument system
- It is useful to systematically combine base logics for applications (e.g. description logic with either defeasible logic or classical logic).