A characterization of collective conflict for defeasible argumentation

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Motivation

• To define a **recursive semantics** (Pollock, 2009) for warranted conclusions in a **general framework** (without levels of strength)
  - A **collective** (non-binary) notion of conflict between arguments
  - **Direct and indirect consistency** without distinguishing between direct and indirect conflicts between arguments

• To **extend the recursive semantics** to an argumentation framework with **levels of preference** (defeasibility) by providing a level-wise definition of warranted and blocked conclusions (Alsinet et al., 2008)

• To **specialize the warrant recursive semantics** for the particular framework of Defeasible Logic Programming (García and Simari, 2004), we refer to this formalism as **RP-DeLP** (Recursive Possibilistic Defeasible Logic Programming)
General defeasible argumentation framework

- **Language** based on a propositional logic \((\mathcal{L}, \vdash)\) with a special symbol \(\bot\) for contradiction

- **Knowledge base**: \((\Pi, \Delta, \Sigma)\), where \(\Pi, \Delta, \Sigma \subseteq \mathcal{L}\) and \(\Pi \not\vdash \bot\)
  - \(\Pi\): strict knowledge (true formulas)
  - \(\Delta\): defeasible knowledge (formulas for which we have reasons to believe they are true)
  - \(\Sigma\): Set of formulas over which arguments can be built

- An **argument** for a formula \(\varphi \in \Sigma\) is is a pair \(\mathcal{A} = \langle A, \varphi \rangle\), with \(A \subseteq \Delta\) such that:
  1. \(\Pi \cup A \not\vdash \bot\), and
  2. \(A\) is minimal (w.r.t. set inclusion) such that \(\Pi \cup A \vdash \varphi\).

- **Defeasible argument**: \(\langle A, \varphi \rangle\) such that \(A \neq \emptyset\)

- A formula \(\varphi \in \Sigma\) is **justifiable** if there exists \(A \subseteq \Delta\) such that \(\langle A, \varphi \rangle\) is an argument
General defeasible argumentation framework

- The notion of subargument is referred to defeasible arguments and expresses an incremental notion of proof between arguments

Example: \((\Pi, \Delta, \Sigma) = (\{r\}, \{r \rightarrow p \land q\}, \{p, q, p \land q\})\)

- \(A_1 = \langle A, p \rangle\), \(A_2 = \langle A, q \rangle\) and \(A_3 = \langle A, p \land q \rangle\) are arguments for different formulas with support \(A = \{r \rightarrow p \land q\}\)

- Subargument relation: \(A_3 \sqsubseteq A_1\) and \(A_3 \sqsubseteq A_2\) since \(p \land q \vdash p\) and \(p \land q \vdash q\)

- No subargument relation: \(A_1 \not\sqsubseteq A_3\) and \(A_2 \not\sqsubseteq A_3\) since \(p \not\vdash p \land q\) and \(q \not\vdash p \land q\)
Collective conflict: motivation

**Aim:** To find a set of justifiable formulas consistent w.r.t. the strict knowledge

**Example:** \( \mathcal{P} = (\Pi, \Delta, \Sigma) \) with

\[
\Pi = \{a \land b \rightarrow \neg p\}, \quad \Delta = \{a, b, p\} \quad \text{and} \quad \Sigma = \{a, b, p, \neg p\}.
\]

- \( \langle \{p\}, p \rangle, \langle \{b\}, b \rangle, \langle \{a\}, a \rangle \) are arguments that justify \( p, b \) and \( a \), respectively.

- **No binary conflict relation:** \( \Pi \cup \{a, b\} \nvdash \bot \), \( \Pi \cup \{a, p\} \nvdash \bot \) and \( \Pi \cup \{b, p\} \nvdash \bot \)

- **Collective conflict relation:** \( \Pi \cup \{a, b, p\} \vdash \bot \)

- \( a, b \) and \( p \) are (collectively) inconsistent w.r.t. \( \Pi \)
Warrant semantics: general framework

Input: a KB $\mathcal{P} = (\Pi, \Delta, \Sigma)$

Output: a pair ($Warr, Block$) such that

- An argument $\langle A, \varphi \rangle$ is either warranted or blocked whenever each subargument is warranted; then, it is eventually warranted if $\varphi$ is not involved in any conflict, otherwise it is blocked
- $\Pi \cup Warr \not\vdash \bot$
- $\Pi \cup Block \vdash \bot$
- $Warr = s-Warr \cup d-Warr$ with
  - $s-Warr = \{ \varphi \mid \Pi \vdash \varphi \} \cap \Sigma$
  - $d-Warr$ and $Block$ are required to satisfy the following recursive constraints
Warrant semantics: general framework

Input: a KB $\mathcal{P} = (\Pi, \Delta, \Sigma)$

Output: a pair $(\text{Warr}, \text{Block})$ where $\text{Warr} = s\text{-Warr} \cup d\text{-Warr}$

• Recursive definition of $d\text{-Warr}$ and $\text{Block}$:

  - A defeasible argument $\langle A, \varphi \rangle$ is valid (not rejected) if all subarguments are warranted

  - For every valid argument $\langle A, \varphi \rangle$

    • $\varphi \in d\text{-Warr}$ whenever there does not exist a set of valid arguments $G$ such that

      (i) Arguments in $G$ do not depend on $\langle A, \varphi \rangle$

      (ii) $G$ and $\langle A, \varphi \rangle$ are inconsistent w.r.t. $\Pi$

    • otherwise, $\varphi \in \text{Block}$
Warrant semantics: example

\(\mathcal{P} = (\Pi, \Delta, \Sigma)\), with \(\Pi = \{a \rightarrow y, b \land c \rightarrow \neg y\}\), \(\Delta = \{a, b, c, \neg c\}\) and \(\Sigma = \{a, b, c, \neg c, y, \neg y\}\)

- \(s\)-\(Warr = \emptyset\)
- Valid arguments: \(\langle\{a\}, a\rangle\), \(\langle\{b\}, b\rangle\), \(\langle\{c\}, c\rangle\) and \(\langle\{\neg c\}, \neg c\rangle\)
- Each valid argument is either warranted or blocked:
  \(\Pi \cup \{a, b, c\} \vdash \bot \implies a, b \text{ and } c \text{ are blocked conclusions}\)
  \(\Pi \cup \{c, \neg c\} \vdash \bot \implies c \text{ and } \neg c \text{ are blocked conclusions}\)
- Output: \(Warr = \emptyset\) and \(Block = \{a, b, c, \neg c\}\)
- Intuition: every conclusion in \(Block\) is (individually) valid, however all together are contradictory w.r.t. \(\Pi\)
- Remark: \(y, \neg y \notin Block\) since arguments \(\langle\{a\}, y\rangle\) and \(\langle\{b, c\}, \neg y\rangle\) depend on \(a, b \text{ and } c \notin Warr\)
Warrant semantics: closure property (general framework)

Let \((Warr, Block)\) be an output for \(P = (\Pi, \Delta, \Sigma)\).

If \(\Pi \cup Warr \vdash \varphi\) then \(\varphi \in Warr\) whenever

1. \(\varphi \in \Sigma\)
2. there exits a valid argument for \(\varphi\)

Example: \(P = (\Pi, \Delta, \Sigma)\) with \(\Pi = \{a \land b \rightarrow y\}\),
\(\Delta = \{s \rightarrow a, \lnot s \rightarrow b, s, \lnot s\}\) and \(\Sigma = \{a, b, y\}\)

Output: \(Warr = \{a, b\}\) and \(Block = \emptyset\)

Problem: \(\Pi \cup Warr \vdash y\) and \(y \in \Sigma\), however \(y \notin Warr\)

Intuition: there does not exist a valid argument for \(y\) since the proof of \(y\) is based on \(a\) and \(b\) which are respectively based on \(s\) and \(\lnot s\)

Solution: to extend \(\Sigma = \{a, b, y, s, \lnot s\}\) \(\Rightarrow\)
\(Warr = \emptyset\) and \(Block = \{s, \lnot s\}\)
Introducing a preference ordering on arguments

- **Stratified knowledge base**: $(\Pi, \Delta, \preceq, \Sigma)$ such that $\preceq$ is a total pre-order on the set of defeasible formulas $\Delta$ representable by a necessity measure $N : \mathcal{L} \rightarrow [0, 1]$

$$\varphi \preceq \psi \text{ iff } N(\varphi) \leq N(\psi)$$

1. $N(\top) = 1$, $N(\bot) = 0$,
2. $N(\varphi \land \psi) = \min(N(\varphi), N(\psi))$
3. $N(\varphi) = 1$ iff $\Pi \vdash \varphi$

- **Strength of an argument**:

$$s(\langle A, \varphi \rangle) = 1 \text{ if } A = \emptyset, \text{ and }$$

$$s(\langle A, \varphi \rangle) = \min\{N(\psi) \mid \psi \in A\}, \text{ otherwise.}$$
Warrant semantics: levels of strength

Input: \((\Pi, \Delta, \preceq, \Sigma)\) such that \(1 > \alpha_1 > \ldots > \alpha_p \geq 0\) are the strengths of defeasible arguments

Output: a pair \((Warr, Block)\) such that

- \(Warr = s-Warr \cup d-Warr\)
- \(s-Warr = \{\varphi \mid \Pi \vdash \varphi\} \cap \Sigma\)
- \(d-Warr = \{d-Warr(\alpha_1), \ldots, d-Warr(\alpha_p)\}\), where \(d-Warr(\alpha_i)\) is the set of warranted conclusions with strength \(\alpha_i\)
- \(Block = \{Block(\alpha_1), \ldots, Block(\alpha_p)\}\), where \(Block(\alpha_i)\) is the set of blocked conclusions with strength \(\alpha_i\)

Extension: a level-wise construction of the sets of conclusions \(Warr(\alpha_i)\) and \(Block(\alpha_i)\)
Warrant semantics: levels of strength

Recursive definition of $Warr(\alpha_i)$ and $Block(\alpha_i)$:

- A defeasible argument $\langle A, \varphi \rangle$ of strength $\alpha_i$ is valid (not rejected) if
  
  1. all subarguments are warranted
  2. $\langle A, \varphi \rangle$ is consistent w.r.t. $\Pi$ and $\cup_{\beta > \alpha_i} d-Warr(\beta)$
  3. $\varphi \notin \cup_{\beta > \alpha_i} d-Warr(\beta)$
  4. $\varphi \notin \cup_{\beta > \alpha_i} Block(\beta)$
  5. $\{\varphi, \psi\} \not\vdash \bot$ for all $\psi \in \cup_{\beta > \alpha_i} Block(\beta)$

- For every valid argument $\langle A, \varphi \rangle$ of strength $\alpha_i$
  
  - $\varphi \in d-Warr(\alpha_i)$ whenever there does not exist a set $G$ of valid arguments of strength $\alpha_i$ such that
    
    (i) Arguments in $G$ do not depend on $\langle A, \varphi \rangle$
    (ii) $G$ and $\langle A, \varphi \rangle$ are inconsistent w.r.t. $\Pi$ and $\cup_{\beta > \alpha_i} d-Warr(\beta)$

  - otherwise, $\varphi \in Block(\alpha_i)$
Levels of strength: example

$\Pi = \{ a \land b \rightarrow \neg p \}$, $\Delta = \{ a, b, p \}$ and $\Sigma = \{ a, b, p, \neg p \}$ extended with two levels of defeasibility:

$$\{ a, b \} \prec p$$

$$\alpha_2 < \alpha_1 < 1$$

- $s$-$Warr = \emptyset$
- Level $\alpha_1$: one valid argument $\langle \{ p \}, p \rangle$
  
  $d$-$Warr(\alpha_1) = \{ p \}$ and $Block(\alpha_1) = \emptyset$

- Level $\alpha_2$:
  
  - Two valid arguments: $\langle \{ a \}, a \rangle$ and $\langle \{ b \}, b \rangle$
  - $\Pi \cup d$-$Warr(\alpha_1) \cup \{ a, b \} \vdash \bot \Rightarrow a$ and $b$ are blocked
  - $a, b \notin d$-$Warr(\alpha_2) \Rightarrow \langle \{ a, b \}, \neg p \rangle$ is not valid $\Rightarrow \neg p$ is rejected

  $$d$-$Warr(\alpha_2) = \emptyset \text{ and } Block(\alpha_2) = \{ a, b \}$$

- Output: $Warr = \{ p \}$ and $Block = \{ a, b \}$
Levels of strength: example

Π = \{a \to y, b \land c \to \neg y\}, \Delta = \{a, b, c, \neg c\} and 
\Sigma = \{a, b, c, \neg c, y, \neg y\} extended with three levels of defeasibility:

\neg c \prec c \prec \{a, b\}

\alpha_3 < \alpha_2 < \alpha_1 < 1

- **s-Warr** = \emptyset
- **Level** \alpha_1: we have not only a, b and y with valid arguments not generating conflict, but also \langle\{a, b\}, \neg c\rangle
  
  \text{d-Warr}(\alpha_1) = \{a, b, y, \neg c\} and \text{Block}(\alpha_1) = \emptyset

- **Level** \alpha_2:
  
  - \Pi \cup \text{d-Warr}(\alpha_1) \cup \{c\} \vdash \bot \Rightarrow c \text{ is rejected}
  
  - \langle\{b, c\}, \neg y\rangle is based on c \notin \text{d-Warr}(\alpha_1) \Rightarrow y \text{ is rejected}

  \text{d-Warr}(\alpha_2) = \emptyset and \text{Block}(\alpha_2) = \emptyset

- **Level** \alpha_3: \neg c \in \text{d-Warr}(\alpha_1) \Rightarrow

  \text{d-Warr}(\alpha_3) = \emptyset and \text{Block}(\alpha_3) = \emptyset
A particular case: Recursive Possibilistic Defeasible Logic Programming (RP-DeLP)

RP-DeLP program: \((\Pi, \Delta, \preceq, \Sigma)\) over the logic \((\mathcal{L}_R, \vdash_R)\), where

- \(\mathcal{L}_R\) consist of atoms \(p, q, \ldots\) extended with a (negated) atom \(\sim p\) for each original atom \(p\)
- Formulas of \(\mathcal{L}_R\) are of the form \(Q \leftarrow P_1 \land \ldots \land P_k\), where \(Q, P_1, \ldots, P_k\) are literals (atoms \(p\) and \(\sim p\))
- \(\vdash_R\) is defined by instances of the modus ponens rule of the form: \(\{Q \leftarrow P_1 \land \ldots \land P_k, P_1, \ldots, P_k\} \vdash_R Q\)
- \(\Sigma\) consists of the set of all literals of \(\mathcal{L}_R\)

Indirect consistency and closure w.r.t. the strict knowledge: Let \((Warr, Block)\) be an output for a RP-DeLP program

- \(\Pi \cup Warr \not\vdash_R \bot\)
- If \(\Pi \cup Warr \vdash_R Q\) then \(Q \in Warr\)
RP-DeLP programs with unique output

- **Circular definitions** of warranty that emerge when considering conflicts among arguments $\Rightarrow$ The output $(Warr, Block)$ for a RP-DeLP program is not unique

- Circular definitions of warranty are characterized by means of **warrant dependency graphs** of RP-DeLP programs

- **Warrant dependency graph**: represents conflict and support dependences among arguments w.r.t. the strict knowledge and a set of warranted conclusions

- A pair $(Warr, Block)$ is the unique output for a RP-DeLP program iff for all literal $L \in Warr$ there is no cycle in the warrant dependency graph for $L$ w.r.t. $Warr$
Example: an empty set of strict clauses and one defeasibility level \( \alpha_1 \) for \( \Delta = \{p, q, \sim p \leftarrow q, \sim q \leftarrow p\} \)

- \( s\text{-}Warr = \emptyset \)

- Level \( \alpha_1 \):
  - Two valid arguments: \( \langle \{p\}, p \rangle \) and \( \langle \{q\}, q \rangle \)
  - Two (possible) valid arguments (support relations with \( p \) and \( q \)): \( \langle \{q, \sim p \leftarrow q\}, \sim p \rangle \) and \( \langle \{p, \sim q \leftarrow p\}, \sim q \rangle \)
  - Two (possible) conflicts at level \( \alpha_1 \):
    \( \{p, \sim p\} \vdash_R \bot \) and \( \{q, \sim q\} \vdash_R \bot \)
  - \( \Rightarrow \) A cycle at the warrant dependency graph:

- Two outputs:
  - \( \text{Output}_1 = (\text{Warr}_1, \text{Block}_1) \) with
    \( \text{Warr}_1 = \{p\} \) and \( \text{Block}_1 = \{q, \sim q\} \)
  - \( \text{Output}_2 = (\text{Warr}_2, \text{Block}_2) \) with
    \( \text{Warr}_2 = \{q\} \) and \( \text{Block}_2 = \{p, \sim p\} \)
Example: $\mathcal{P} = (\Pi, \Delta)$, with $\Pi = \{y, \sim y \leftarrow p \land r, \sim y \leftarrow q \land s\}$ and one defeasibility level $\alpha_1$ for $\Delta = \{p, q, r \leftarrow q, s \leftarrow p\}$

- $s$-$Warr = \{y\}$
- Level $\alpha_1$:
  - Two valid arguments: $\langle \{p\}, p \rangle$ and $\langle \{q\}, q \rangle$
  - Two (possible) valid arguments (support relations with $p$ and $q$): $\langle \{q, r \leftarrow q\}, r \rangle$ and $\langle \{p, s \leftarrow p\}, s \rangle$
  - Two (possible) conflicts at level $\alpha_1$:
    $\Pi \cup s$-$Warr \cup \{p, r\} \vdash_R \bot$ and $\Pi \cup s$-$Warr \cup \{q, s\} \vdash_R \bot$
  - ⇒ A cycle at the warrant dependency graph:

- Two outputs:
  - Output$_1 = (Warr_1, Block_1)$ with $Warr_1 = \{p\}$ and $Block_1 = \{q, s\}$
  - Output$_2 = (Warr_2, Block_2)$ with $Warr_2 = \{q\}$ and $Block_2 = \{p, r\}$
Conclusions

• A recursive semantics for determining the warranty status of arguments in defeasible argumentation
  - A general defeasible argumentation framework based on a propositional logic
  - Levels of defeasibility $\Rightarrow$ Strength of arguments
  - RP-DeLP

• A collective notion of conflict

• Direct and indirect consistency w.r.t. the strict knowledge

• Circular definition of warranty $\Rightarrow$ Warrant dependency graph of a RP-DeLP program

• Not based on dialectical trees $\Rightarrow$ It is not necessary to explicitly compute all the possible arguments for a given literal to check whether it is warranted

• RP-DeLP programs with unique output: implementation with a worst-case complexity in $P^{NP}$