Dialectical Proofs for Constrained Argumentation

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Overview

1. Argumentation Frameworks
   - Dung’s Argumentation Framework
   - Constrained Argumentation Framework

2. Constrained dialectical proofs
   - Dialectical framework
   - Definition of constrained dialectical proofs
   - Computation

3. Conclusion
[Dung95] An argumentation framework is a pair $\langle A, R \rangle$ where:

- $A$ is a set of arguments
- $R \subseteq A \times A$ represents a notion of attack

Can be represented as a directed graph
Argumentation framework - Definition

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A subset \( S \subseteq A \) is **admissible** if:

1. **S is conflict-free**: there are not two arguments in \( S \) such that one attacks the other, and
2. **S defends all its elements**: any argument \( y \in A \setminus S \) that attacks \( x \in S \) is attacked by some \( z \in S \).

\( S \) is a **preferred extension** iff it is maximal w.r.t. \( \subseteq \) among the set of admissible sets.

**Example**

Two of the preferred extensions: \( \{a, d, f, h, j\} \) and \( \{a, d, e, g, j\} \)
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A subset $S \subseteq \mathcal{A}$ is admissible if:

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[Coste-Marquis et al. 06] A Constrained Argumentation Framework (CAF) is a triple $\langle A, R, C \rangle$ where:

- $A$ is a set of arguments
- $R \subseteq A \times A$ represents a notion of attack
- $C$ is a formula from $PROP_A$ (propositional language defined in the usual inductive way from the set $A$) which represents a constraint

Example:

$$C = (k \iff d) \land ((d \implies (f \lor g)) \lor (\neg d \implies \neg f))$$
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A subset $S \subseteq \mathcal{A}$ is $\mathcal{C}$-admissible iff:

- $S$ is admissible for $\langle \mathcal{A}, \mathcal{R} \rangle$, and
- $S$ satisfies $\mathcal{C}$, that is, $\hat{S} = \{a \mid a \in S\} \cup \{\neg a \mid a \in \mathcal{A} \setminus S\}$ is a model of $\mathcal{C}$

$S$ is a preferred $\mathcal{C}$-extension iff it is maximal w.r.t. $\subseteq$ among the set of $\mathcal{C}$-admissible sets.

For each $\mathcal{C}$-admissible set $X$ of $CAF$, there exists a preferred $\mathcal{C}$-extension $E$ of $CAF$ such that $X \subseteq E$. 

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Constrained argumentation framework - Semantics

- Dung’s Argumentation Framework
- Constrained Argumentation Framework
- Conclusion

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Argumentation Frameworks
Constrained dialectical proofs
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Constrained argumentation framework - Properties

- Generalizes other argumentation frameworks and semantics [Coste-Marquis et al. 06], e.g.:
  - Dung’s argumentation framework and the preferred semantics
    - Let $AF = \langle A, R \rangle$ be an argumentation framework. Let $CAF = \langle A, R, C \rangle$ be a constrained argumentation framework where $C$ is any valid formula. Then the preferred extensions of $AF$ are the preferred $C$-extensions of $CAF$.
  - Cayrol and Lagasquie-Schiex’s bipolar argumentation framework and the weakly c-preferred semantics
  - ...
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Credulous acceptance problem under the $\mathcal{C}$-preferred semantics:

Given a CAF $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$,

is a given set $S \subseteq \mathcal{A}$ included in at least one preferred $\mathcal{C}$-extension of CAF?

Example

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Is $\{e, k\}$ included in at least one preferred $\mathcal{C}$-extension?
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Adaptation of [Cayrol et al. 03] definitions:

**Definition**

Let \( \mathcal{A} \) be a set of arguments. Let \( _{\empty} \) be an “empty” argument. A **dialogue** is a finite sequence

\[
d = \langle a_0.a_1.a_2 \ldots a_n \rangle
\]

of arguments from \( \mathcal{A}^\_ = \mathcal{A} \cup \{_{\empty}\} \).

The **player** of \( a_i, i \in \{0 \ldots n\} \), in \( d \) is:

- **PRO** if \( i \) is even and
- **OPP** if \( i \) is odd
Dialectical framework

Definition

Let $\phi : A^* \to 2^A$ a function called legal-move function. A $\phi$-dialogue for a set of arguments $S \subseteq A$ is a dialogue $d$ such that:

1. $\forall i \geq 0, a_i \in \phi(d_i)$, and
2. $S$ is included in PRO($d$), the set of arguments played by PRO in $d$. 
Specific legal-move function $\phi_C$ defined to answer the credulous acceptance problem under the $C$-preferred semantics.

**Definition**

Let $\langle \mathcal{A}, \mathcal{R}, \mathcal{C} \rangle$ be a constrained argumentation framework and $S \subseteq \mathcal{A}$ be a set of arguments.

A $\phi_C$-proof for $S$ is a $\phi_C$-dialogue $d$ for $S$ such that:

1. either $d$ is empty or $d$ ends with the empty argument, and
2. the set of arguments played by PRO in $d$ satisfies $\mathcal{C}$

We say that $d$ is won by PRO.
Constrained dialectical proofs

⇒ Specific legal-move function $\phi_C$ defined to answer the credulous acceptance problem under the $C$-preferred semantics.

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Constrained dialectical proofs

Proposition (Correctness and Completeness)

Let $CAF = \langle A, R, C \rangle$ be a constrained argumentation framework.

- If $d$ is a $\phi_C$-proof for a set of arguments $S \subseteq A$, then the set of arguments played by PRO in $d$ is a $C$-admissible set of $CAF$ that contains $S$.

- If a set of arguments $S$ is included in a $C$-admissible set of $CAF$ then there exists a $\phi_C$-proof for $S$. 
Constrained dialectical proofs - Example

Example

\[ C = \left( k \leftrightarrow d \right) \land \left( d \Rightarrow (f \lor g) \right) \lor \left( \neg d \Rightarrow \neg f \right) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

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\begin{align*}
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d_1 &= \langle e \rangle & \phi_C(d_1) &= \{c\} \\
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### Example

**Constrained dialectical proofs - Example**

![Diagram](attachment:image.png)

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Constrained dialectical proofs - Example

Definition of constrained dialectical proofs

Example

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    d_4 &= \langle e.c.a._\rangle & S \nsubseteq PRO(d_4) \text{ then } \phi_c(d_4) &= \{k\} \\
    d_5 &= \langle e.c.a._.k \rangle & \phi_c(d_5) &= \{\_\}
\end{align*}
\]
Constrained dialectical proofs - Example

Example

\[ C = (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_0 = \langle \rangle \quad \phi_C(d_0) = \{e, k\} \]
\[ d_1 = \langle e \rangle \quad \phi_C(d_1) = \{c\} \]
\[ d_2 = \langle e.c \rangle \quad \phi_C(d_2) = \{a\} \]
\[ d_3 = \langle e.c.a \rangle \quad \phi_C(d_3) = \{\_\} \]
\[ d_4 = \langle e.c.a._.\_ \rangle \quad S \not\subseteq \text{PRO}(d_4) \text{ then } \phi_C(d_4) = \{k\} \]
\[ d_5 = \langle e.c.a._.\_.k \rangle \quad \phi_C(d_5) = \{\_\} \]
Constrained dialectical proofs - Example

Example

\[ C = (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_0 = \langle \rangle \quad \phi_C(d_0) = \{e, k\} \]
\[ d_1 = \langle e \rangle \quad \phi_C(d_1) = \{c\} \]
\[ d_2 = \langle e.c \rangle \quad \phi_C(d_2) = \{a\} \]
\[ d_3 = \langle e.c.a \rangle \quad \phi_C(d_3) = \{\_\} \]
\[ d_4 = \langle e.c.a.\_ \rangle \quad S \not\subseteq \text{PRO}(d_4) \text{ then } \phi_C(d_4) = \{k\} \]
\[ d_5 = \langle e.c.a.\_.k \rangle \quad \phi_C(d_5) = \{\_\} \]
Constrained dialectical proofs - Example

Example

\[ C = (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._.k._.\rangle \]
\[ d_7 = \langle e.c.a._.k._.g.\rangle \]
\[ d_8 = \langle e.c.a._.k._.g._.\rangle \]
\[ d_9 = \langle e.c.a._.k._.g._.d\rangle \]

\( S \subseteq \text{PRO}(d_6) \) but \( \overline{\text{PRO}(d_6)} \not\models C \)

so \( \phi_C(d_6) = \{d, i, h, g\} \)

\( S \subseteq \text{PRO}(d_7) \) but \( \overline{\text{PRO}(d_7)} \not\models C \)

so \( \phi_C(d_7) = \{\_\} \)

\( S \subseteq \text{PRO}(d_8) \) but \( \overline{\text{PRO}(d_8)} \not\models C \)

so \( \phi_C(d_8) = \{d, i\} \)

\( S \subseteq \text{PRO}(d_9) \) but \( \overline{\text{PRO}(d_9)} \not\models C \)

so \( \phi_C(d_9) = \{\_\} \)
Constrained dialectical proofs - Example

Example

\[ C = (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._.k._.\rangle \quad S \subseteq \text{PRO}(d_6) \text{ but } \widehat{\text{PRO}}(d_6) \not\models C \]

so \( \phi_C(d_6) = \{d, i, h, g\} \)

\[ d_7 = \langle e.c.a._.k._.g.\rangle \quad \phi_C(d_7) = \{\_\} \]

\[ d_8 = \langle e.c.a._.k._.g._.\rangle \quad S \subseteq \text{PRO}(d_8) \text{ but } \widehat{\text{PRO}}(d_8) \not\models C \]

so \( \phi_C(d_8) = \{d, i\} \)

\[ d_9 = \langle e.c.a._.k._.g._.d.\rangle \quad \phi_C(d_9) = \{\_\} \]
Constrained dialectical proofs - Example

Example

\[ C = (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._.k._.\rangle \]
\[ d_7 = \langle e.c.a._.k._.g.\rangle \]
\[ d_8 = \langle e.c.a._.k._.g._.\rangle \]
\[ d_9 = \langle e.c.a._.k._.g._.d.\rangle \]

\[ S \subseteq \text{PRO}(d_6) \text{ but } \overline{\text{PRO}(d_6)} \not\models C \]
so \( \phi_C(d_6) = \{d, i, h, g\} \)

\[ \phi_C(d_7) = \{\_\} \]
\[ \phi_C(d_8) = \{\_\} \]

\[ S \subseteq \text{PRO}(d_8) \text{ but } \overline{\text{PRO}(d_8)} \not\models C \]
so \( \phi_C(d_8) = \{d, i\} \)

\[ \phi_C(d_9) = \{\_\} \]
Constrained dialectical proofs - Example

Example

\[ C = (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._.k._.\rangle \quad S \subseteq PRO(d_6) \text{ but } PRO(d_6) \not\models C \]

so \( \phi_C(d_6) = \{d, i, h, g\} \)

\[ d_7 = \langle e.c.a._.k._.g.\rangle \quad \phi_C(d_7) = \{\_\} \]

\[ d_8 = \langle e.c.a._.k._.g._.\rangle \quad S \subseteq PRO(d_8) \text{ but } PRO(d_8) \not\models C \]

so \( \phi_C(d_8) = \{d, i\} \)

\[ d_9 = \langle e.c.a._.k._.g._.d\rangle \quad \phi_C(d_9) = \{\_\} \]
Constrained dialectical proofs - Example

Example

\[
C = (k \leftrightarrow d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f))
\]

Is \(S = \{e, k\}\) included in at least one preferred \(C\)-extension?

\[
d_6 = \langle e.c.a._.k._.\rangle
\]
\[
S \subseteq \text{PRO}(d_6) \text{ but } \overline{\text{PRO}}(d_6) \not\models C
\]
so \(\phi_C(d_6) = \{d, i, h, g\}\)

\[
d_7 = \langle e.c.a._.k._.g.\rangle
\]
\[
\phi_C(d_7) = \{\_\}
\]

\[
d_8 = \langle e.c.a._.k._.g._.\rangle
\]
\[
S \subseteq \text{PRO}(d_8) \text{ but } \overline{\text{PRO}}(d_8) \not\models C
\]
so \(\phi_C(d_8) = \{d, i\}\)

\[
d_9 = \langle e.c.a._.k._.g._.d.\rangle
\]
\[
\phi_C(d_9) = \{\_\}\]
Constrained dialectical proofs - Example

Example

\[ C = (k \Leftrightarrow d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._k._\rangle \quad S \subseteq \text{PRO}(d_6) \text{ but } \text{PRO}(d_6) \not\models C \]

so \( \phi_C(d_6) = \{d, i, h, g\} \)

\[ d_7 = \langle e.c.a._k._.\rangle \quad \phi_C(d_7) = \{\_\} \]

\[ d_8 = \langle e.c.a._k._.g.\rangle \quad S \subseteq \text{PRO}(d_8) \text{ but } \text{PRO}(d_8) \not\models C \]

so \( \phi_C(d_8) = \{d, i\} \)

\[ d_9 = \langle e.c.a._k._.g.d.\rangle \quad \phi_C(d_9) = \{\_\} \]
**Constrained dialectical proofs - Example**

**Example**

\[
\begin{align*}
C &= (k \leftrightarrow d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f))
\end{align*}
\]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[
\begin{align*}
d_6 &= \langle e.\,c.\,a.\,\_,\,k.\,\_,\,\rangle & S \subseteq \text{PRO}(d_6) \text{ but } \widehat{\text{PRO}}(d_6) \not\models C \\
s&\quad \text{so } \phi_C(d_6) = \{d, i, h, g\} \\
d_7 &= \langle e.\,c.\,a.\,\_,\,k.\,\_,\,g.\,\rangle & \phi_C(d_7) = \{\_\} \\
d_8 &= \langle e.\,c.\,a.\,\_,\,k.\,\_,\,g.\,\_,\,\rangle & S \subseteq \text{PRO}(d_8) \text{ but } \widehat{\text{PRO}}(d_8) \not\models C \\
s&\quad \text{so } \phi_C(d_8) = \{d, i\} \\
d_9 &= \langle e.\,c.\,a.\,\_,\,k.\,\_,\,g.\,\_,\,d\,\rangle & \phi_C(d_9) = \{\_\}
\end{align*}
\]
Constrained dialectical proofs - Example

Example

\[ C = (k \Leftrightarrow d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{ e, k \} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._k._. \rangle \]
\[ S \subseteq \text{PRO}(d_6) \text{ but } \text{PRO}(d_6) \not\models C \]
so \( \phi_C(d_6) = \{ d, i, h, g \} \)

\[ d_7 = \langle e.c.a._k._.g. \rangle \]
\[ \phi_C(d_7) = \{ \_ \} \]

\[ d_8 = \langle e.c.a._k._.g._. \rangle \]
\[ S \subseteq \text{PRO}(d_8) \text{ but } \text{PRO}(d_8) \not\models C \]
so \( \phi_C(d_8) = \{ d, i \} \)

\[ d_9 = \langle e.c.a._k._.g._.d \rangle \]
\[ \phi_C(d_9) = \{ \_ \} \]
Constraint dialectical proofs - Example

Example

\[ C = (k \leftrightarrow d') \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._.k._.\rangle \quad S \subseteq \text{PRO}(d_6) \text{ but PRO}(d_6) \not\models C \]

so \( \phi_C(d_6) = \{d, i, h, g\} \)

\[ d_7 = \langle e.c.a._.k._.g.\rangle \quad \phi_C(d_7) = \{\_\} \]

\[ d_8 = \langle e.c.a._.k._.g._.\rangle \quad S \subseteq \text{PRO}(d_8) \text{ but PRO}(d_8) \not\models C \]

so \( \phi_C(d_8) = \{d, i\} \)

\[ d_9 = \langle e.c.a._.k._.g._.d\rangle \quad \phi_C(d_9) = \{\_\} \]
Example

\[ C = (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._.k._.\rangle \]

\[ d_7 = \langle e.c.a._.k._.g.\rangle \]

\[ d_8 = \langle e.c.a._.k._.g._.\rangle \]

\[ d_9 = \langle e.c.a._.k._.g._.d\rangle \]

\( S \subseteq \text{PRO}(d_6) \) but \( \overline{\text{PRO}(d_6)} \not\models C \)

so \( \phi_C(d_6) = \{d, i, h, g\} \)

\( S \subseteq \text{PRO}(d_7) \) but \( \overline{\text{PRO}(d_7)} \not\models C \)

so \( \phi_C(d_7) = \{\_\} \)

\( S \subseteq \text{PRO}(d_8) \) but \( \overline{\text{PRO}(d_8)} \not\models C \)

so \( \phi_C(d_8) = \{d, i\} \)

\( S \subseteq \text{PRO}(d_9) \) but \( \overline{\text{PRO}(d_9)} \not\models C \)

so \( \phi_C(d_9) = \{\_\} \)
Constrained dialectical proofs - Example

**Example**

\[ C = (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

- \( d_6 = \langle e.c.a._.k._.\rangle \) \( S \subseteq \text{PRO}(d_6) \) but \( \widehat{\text{PRO}}(d_6) \not\models C \)
  - so \( \phi_C(d_6) = \{d, i, h, g\} \)

- \( d_7 = \langle e.c.a._.k._.g\rangle \) \( \phi_C(d_7) = \{\_\} \)

- \( d_8 = \langle e.c.a._.k._.g._.\rangle \) \( S \subseteq \text{PRO}(d_8) \) but \( \widehat{\text{PRO}}(d_8) \not\models C \)
  - so \( \phi_C(d_8) = \{d, i\} \)

- \( d_9 = \langle e.c.a._.k._.g._.d\rangle \) \( \phi_C(d_9) = \{\_\} \)
Constrained dialectical proofs - Example

Example

\[ C = (k \leftrightarrow d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._k._\rangle \quad \text{S} \subseteq \text{PRO}(d_6) \text{ but } \text{PRO}(d_6) \not\models C \]

so \( \phi_C(d_6) = \{d, i, h, g\} \)

\[ d_7 = \langle e.c.a._k._g._\rangle \quad \phi_C(d_7) = \{\_\} \]

\[ d_8 = \langle e.c.a._k._g._d._\rangle \quad S \subseteq \text{PRO}(d_8) \text{ but } \text{PRO}(d_8) \not\models C \]

so \( \phi_C(d_8) = \{d, i\} \)

\[ d_9 = \langle e.c.a._k._g._d._\rangle \quad \phi_C(d_9) = \{\_\} \]
Constrained dialectical proofs - Example

\[ C = (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._.k._\rangle \quad S \subseteq PRO(d_6) \text{ but } PRO(d_6) \not\models C \]

so \( \phi_C(d_6) = \{d, i, h, g\} \)

\[ d_7 = \langle e.c.a._.k._.g\rangle \quad \phi_C(d_7) = \{\_\} \]

\[ d_8 = \langle e.c.a._.k._.g._\rangle \quad S \subseteq PRO(d_8) \text{ but } PRO(d_8) \not\models C \]

so \( \phi_C(d_8) = \{d, i\} \)

\[ d_9 = \langle e.c.a._.k._.g._.d\rangle \quad \phi_C(d_9) = \{\_\} \]
Constrained dialectical proofs - Example

Example

\[ C = (k \Leftrightarrow d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._.k._. \rangle \quad \text{S \subseteq PRO}(d_6) \text{ but } \widehat{\text{PRO}}(d_6) \not\models C \]

so \( \phi_C(d_6) = \{d, i, h, g\} \)

\[ d_7 = \langle e.c.a._.k._.g. \rangle \quad \phi_C(d_7) = \{\_\} \]

\[ d_8 = \langle e.c.a._.k._.g._. \rangle \quad \text{S \subseteq PRO}(d_8) \text{ but } \widehat{\text{PRO}}(d_8) \not\models C \]

so \( \phi_C(d_8) = \{d, i\} \)

\[ d_9 = \langle e.c.a._.k._.g._.d. \rangle \quad \phi_C(d_9) = \{\_\} \]
Constrained dialectical proofs - Example

Example

$$C = (k \Leftrightarrow d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f))$$

Is $S = \{e, k\}$ included in at least one preferred $C$-extension?

$$d_6 = \langle e.c.a._.k._.\rangle$$

$S \subseteq \text{PRO}(d_6)$ but $\widehat{\text{PRO}}(d_6) \not\models C$

so $\phi_C(d_6) = \{d, i, h, g\}$

$$d_7 = \langle e.c.a._.k._.g.\rangle$$

$\phi_C(d_7) = \{\_\}$

$$d_8 = \langle e.c.a._.k._.g._.\rangle$$

$S \subseteq \text{PRO}(d_8)$ but $\widehat{\text{PRO}}(d_8) \not\models C$

so $\phi_C(d_8) = \{d, i\}$

$$d_9 = \langle e.c.a._.k._.g._.d\rangle$$

$\phi_C(d_9) = \{\_\}$
Constrained dialectical proofs - Example

Example

\[
\begin{align*}
\mathcal{C} &= (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \\
S &= \{e, k\} \text{ included in at least one preferred } \mathcal{C}\text{-extension?}
\end{align*}
\]

\[
\begin{align*}
d_6 &= \langle e.c.a._.k_.\rangle & S &\subseteq \text{PRO}(d_6) \text{ but } \overline{\text{PRO}(d_6)} &\nmid\not\in \mathcal{C} \\
&\text{so } \phi_C(d_6) = \{d, i, h, g\} \\
d_7 &= \langle e.c.a._.k_.g.\rangle & \phi_C(d_7) = \{\_\} \\
d_8 &= \langle e.c.a._.k_.g_.\rangle & S &\subseteq \text{PRO}(d_8) \text{ but } \overline{\text{PRO}(d_8)} \nmid\not\in \mathcal{C} \\
&\text{so } \phi_C(d_8) = \{d, i\} \\
d_9 &= \langle e.c.a._.k_.g_.d\rangle & \phi_C(d_9) = \{\_\}
\end{align*}
\]
Constrained dialectical proofs - Example

Example

\[ C = (k \leftrightarrow d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._.k._.\rangle \quad S \subseteq \text{PRO}(d_6) \quad \text{but} \quad \text{PRO}(d_6) \not\models C \]

so \( \phi_C(d_6) = \{d, i, h, g\} \)

\[ d_7 = \langle e.c.a._.k._.g.\rangle \quad \phi_C(d_7) = \{\_\} \]

\[ d_8 = \langle e.c.a._.k._.g._.\rangle \quad S \subseteq \text{PRO}(d_8) \quad \text{but} \quad \text{PRO}(d_8) \not\models C \]

so \( \phi_C(d_8) = \{d, i\} \)

\[ d_9 = \langle e.c.a._.k._.g._.d.\rangle \quad \phi_C(d_9) = \{\_\} \]
Constrained dialectical proofs - Example

Example

\[ C = (k \Leftrightarrow d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._.k._.\rangle \quad S \subseteq \text{PRO}(d_6) \text{ but } \overline{\text{PRO}(d_6)} \not\models C \]

so \( \phi_C(d_6) = \{d, i, h, g\} \)

\[ d_7 = \langle e.c.a._.k._.g.\rangle \quad \phi_C(d_7) = \{\_\} \]

\[ d_8 = \langle e.c.a._.k._.g.\_\rangle \quad S \subseteq \text{PRO}(d_8) \text{ but } \overline{\text{PRO}(d_8)} \not\models C \]

so \( \phi_C(d_8) = \{d, i\} \)

\[ d_9 = \langle e.c.a._.k._.g._d\rangle \quad \phi_C(d_9) = \{\_\} \]
Constrained dialectical proofs - Example

Example

\[ C = (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_6 = \langle e.c.a._.k._. \rangle \]
\[ S \subseteq \text{PRO}(d_6) \text{ but } \overline{\text{PRO}(d_6)} \not \models C \]
so \( \phi_C(d_6) = \{d, i, h, g\} \)

\[ d_7 = \langle e.c.a._.k._.g. \rangle \]
\[ \phi_C(d_7) = \{\_\} \]

\[ d_8 = \langle e.c.a._.k._.g._. \rangle \]
\[ S \subseteq \text{PRO}(d_8) \text{ but } \overline{\text{PRO}(d_8)} \not \models C \]
so \( \phi_C(d_8) = \{d, i\} \)

\[ d_9 = \langle e.c.a._.k._.g._.d \rangle \]
\[ \phi_C(d_9) = \{\_\} \]
Constrained dialectical proofs - Example

Example

\[ C = (k \Leftrightarrow d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_{10} = \langle e.c.a._.k._.g._.d._.\rangle \quad S \subseteq \text{PRO}(d_{10}), \overset{\text{PRO}(d_{10})}{\models} C \quad \text{so } \phi_C(d_{10}) = \emptyset \]

\( d_{10} \) is a \( \phi_C \)-dialogue won by PRO. 
\( \text{PRO}(d_{10}) = \{e, a, k, g, d\} \) is a \( C \)-admissible set. 
\( \Rightarrow \{e, k\} \) is included into at least one preferred \( C \)-extension
Constrained dialectical proofs - Example

Example

\[ C = (k \Leftrightarrow d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_{10} = \langle e.c.a._.k._.g._.d._. \rangle \quad S \subseteq \text{PRO}(d_{10}), \ \text{PRO}(d_{10}) \models C \]

so \( \phi_C(d_{10}) = \emptyset \)

\( d_{10} \) is a \( \phi_C \)-dialogue won by PRO.

\( \text{PRO}(d_{10}) = \{e, a, k, g, d\} \) is a \( C \)-admissible set.

\( \Rightarrow \) \{e, k\} is included into at least one preferred \( C \)-extension
Constrained dialectical proofs - Example

Example

\[ \begin{align*}
C &= (k \iff d) \land ((d \implies (f \lor g)) \lor (\neg d \implies \neg f)) \\
\text{Is } S &= \{e, k\} \text{ included in at least one preferred } C\text{-extension?}
\end{align*} \]

\[ d_{10} = \langle e.c.a._.k._.g._.d._. \rangle \quad S \subseteq \text{PRO}(d_{10}), \overline{\text{PRO}(d_{10})} \models C \]

so \( \phi_C(d_{10}) = \emptyset \)

\( d_{10} \) is a \( \phi_C \)-dialogue won by PRO.

\( \text{PRO}(d_{10}) = \{e, a, k, g, d\} \) is a \( C \)-admissible set.

\( \Rightarrow \{e, k\} \text{ is included into at least one preferred } C\text{-extension} \)
Constrained dialectical proofs - Example

**Example**

\[ C = (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{ e, k \} \) included in at least one preferred \( C \)-extension?

\[ d_{10} = \langle e . c . a . \_ . k . \_ . g . \_ . d . \_ \rangle \quad S \subseteq \text{PRO}(d_{10}), \overline{\text{PRO}(d_{10})} \models C \]

so \( \phi_C(d_{10}) = \emptyset \)

\( d_{10} \) is a \( \phi_C \)-dialogue won by PRO.

\( \text{PRO}(d_{10}) = \{ e, a, k, g, d \} \) is a \( C \)-admissible set.

\( \Rightarrow \{ e, k \} \) is included into at least one preferred \( C \)-extension
Constrained dialectical proofs - Example

Example

\[ C = (k \iff d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_{10} = \langle e, c, a, \_, k, \_, g, \_, d, \_ \rangle \quad S \subseteq \text{PRO}(d_{10}), \overline{\text{PRO}(d_{10})} \models C \]

so \( \phi_C(d_{10}) = \emptyset \)

\( d_{10} \) is a \( \phi_C \)-dialogue won by PRO.

\( \text{PRO}(d_{10}) = \{e, a, k, g, d\} \) is a \( C \)-admissible set.

\( \Rightarrow \{e, k\} \) is included into at least one preferred \( C \)-extension
Constrained dialectical proofs - Example

Example

\[ C = (k \leftrightarrow d) \land ((d \Rightarrow (f \lor g)) \lor (\neg d \Rightarrow \neg f)) \]

Is \( S = \{e, k\} \) included in at least one preferred \( C \)-extension?

\[ d_{10} = \langle e, c, a, \_ , k, \_ , g, \_ , d, \_ \rangle \quad S \subseteq \text{PRO}(d_{10}), \overline{\text{PRO}(d_{10})} \models C \]

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Constrained dialectical proofs - Example

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Computation by ASP

- **Answer Set Programming:**
  - Simple and readable encoding
  - Well adapted to encode the iterating and alternating roles of pros and cons

- **Computation** of the constrained dialectical proofs:
  - In the ASP solver **ASPeRiX**
  - Program available at
    
    http://www.info.univ-angers.fr/pub/claire/asperix/Argumentation
Constrained argumentation frameworks: generalize other existing frameworks and semantics

Simple and general **dialectical framework**

**Dialectical proofs** for the credulous acceptance problem under the $C$-preferred semantics

**Computation** in ASP